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DATA FOR THE PROBLEM OF EVOLUTION IN MAN.

ON FERTILITY, DURATION OF LIFE AND REPRODUCTIVE SELECTION.

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A. *On the Correlation between Duration of Life and Number of Offspring.*

IN a previous paper on this subject* the data were derived from the published records of New South Wales for the two years 1898 and 1899, and referred only to the deaths of married women. As the experience of the subsequent three years with regard to married women, and that of 1901 and 1902 with regard to married men, are now available†, I have used the complete experience for each sex as the basis of a fresh investigation of the correlation between the duration of life and number of offspring, and as the number of observations is large, viz. 15,548 deaths of married women and 7303 of married men, the results should be of some value in the determination of this subject. In Tables I. and II. are

TABLE I.

Ages of Married Men at Death and Average Number of Offspring (Living and Dead). Experience of New South Wales, 1901 *and* 1902.

Ages at Death	Deaths of Married Men	Total Offspring	Average Offspring
20 and under *25*	43	39	·91
25 " *30*	173	212	1·23
30 " *35*	296	691	2·33
35 " *40*	477	1454	3·05
40 " *45*	574	2329	4·06
45 " *50*	603	2927	4·85
50 " *55*	578	3073	5·32
55 " *60*	642	3908	6·09
60 " *65*	731	4452	6·09
65 " *70*	913	5804	6·36
70 " *75*	849	5651	6·66
75 " *80*	591	3653	6·18
80 " *85*	428	2774	6·48
85 " *90*	267	1747	6·54
90 " *95*	96	640	6·67
95 " *100*	33	218	6·61
100 and over	9	43	4·78
Total	7,303	39,615	5·42

* *Biometrika*, Vol. I. Part I. pp. 34—38.

† Vide "Vital Statistics" of New South Wales for 1900, 1901 and 1902.

contained the complete data, and also the average number of offspring to the married of each sex in each quinquennial age-group at death.

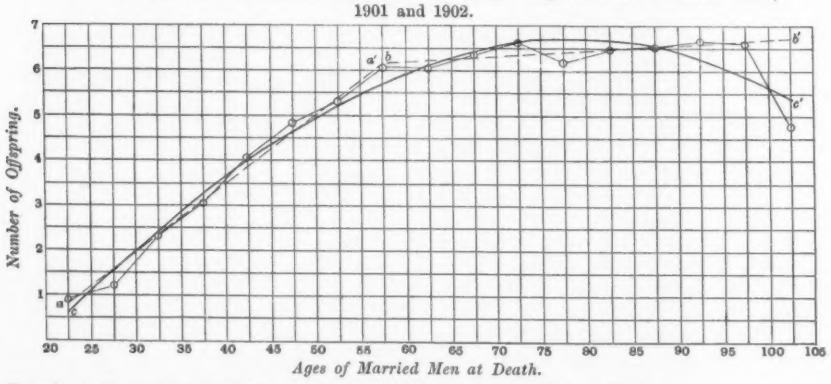
These averages will be found plotted in Figs. 1 and 2. On reference to Fig. 1, it will be seen that the male statistics divide into two well-defined series,

TABLE II.

Ages of Married Women at Death and Number of Offspring (Living and Dead). Experience of New South Wales, 1898 *to* 1902.

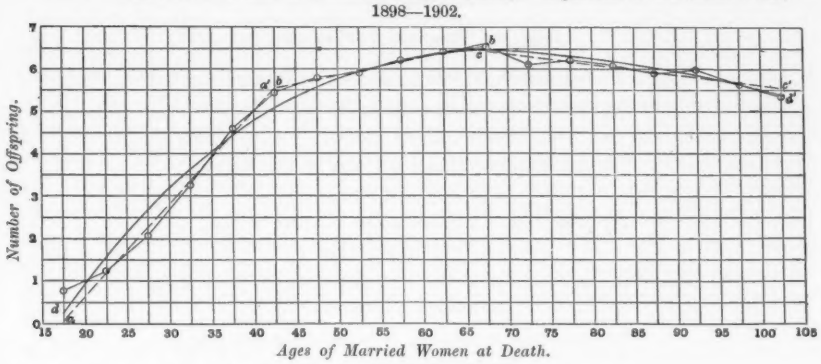
Ages at Death	Deaths of Married Women	Total Offspring	Average Offspring
Under *20*	69	54	·78
20 and under *25*	480	598	1·25
25 „ *30*	937	1970	2·10
30 „ *35*	1131	3711	3·28
35 „ *40*	1311	6004	4·58
40 „ *45*	1101	6011	5·46
45 „ *50*	1040	6048	5·82
50 „ *55*	958	5669	5·92
55 „ *60*	1116	6955	6·23
60 „ *65*	1343	8657	6·45
65 „ *70*	1485	9741	6·56
70 „ *75*	1410	8628	6·12
75 „ *80*	1269	7921	6·24
80 „ *85*	1073	6539	6·10
85 „ *90*	544	3201	5·88
90 „ *95*	204	1225	6·00
95 „ *100*	63	355	5·63
100 and over	14	75	5·36
Total	15,548	83,362	5·36

FIG. 1. Length of Life of Married Men and Size of Family. Experience of New South Wales, 1901 and 1902.



Equation to Regression Straight Line aa', $y = -2{\cdot}6570 + {\cdot}15391x$ } Origin at Birth.
„ „ „ bb', $y = 5{\cdot}39489 + {\cdot}013284x$ } Unit of $x = 1$ year.
„ „ Cubical Parabola cc', $y = 6{\cdot}13673 + {\cdot}34808x - {\cdot}04913x^2 - {\cdot}00078x^3$ { Origin at 62·5 years. Unit of $x = 5$ years.

FIG. 2. Length of Life of Married Women and Size of Family. Experience of New South Wales, 1898—1902.



Equation to Regression Straight Line aa', $y = -3{\cdot}7179 + {\cdot}21729x$ } Origin at Birth.
" " " bb', $y = 3{\cdot}6866 + {\cdot}04335x$ } Unit of $x = 1$ year.
" " " cc', $y = 8{\cdot}1556 - {\cdot}025366x$ }
" " Cubical Parabola dd', $y = 6{\cdot}29315 + {\cdot}15595x - {\cdot}04839x^2 + {\cdot}00199x^3$ { Origin at 60 years. Unit of $x = 5$ years.

(a) from age period 20—25 to 50—55, and (b) from 55—60 to 95—100. The observations beyond age 100 are too few to attach much weight to the results. From Fig. 2 it is seen that the female statistics divide into three well-defined series, (a) from age group 15—20 to 40—45, (b) from 40—45 to 65—70, and (c) from 65—70 to 100—105. These divisions in each sex are practically in agreement with those of Professor Pearson's English and American Quaker families*—more especially the former—although owing to the comparative paucity of data, his series are not so well defined. The New South Wales data for ages beyond 75 are sufficiently extensive to furnish reliable results up to age 100.

The correlation between the duration of life and number of offspring and the regression straight lines have been determined for each of the five series and the general results are set forth in Tables III. and IV.

TABLE III. *General Results.*

Parent	Period of Life	Mean Age at Death (years)	Mean Size of Family	Standard Deviation		Correlation between Duration of Life and Fertility	Regression	Probable error of y (Size of Family)
				Age at Deaths†	Size of Family			
Father	Under *60*	45·3424	4·3216	1·8434	1·4266	·99436 ± ·00013	+ ·76954	± ·1020
"	*60* and over	70·90975	6·3369	1·9050	·2304	·54915 ± ·00698	+ ·06642	± ·1299
Mother	Under *45*	33·6208	3·6484	1·3226	1·4436	·99541 ± ·00009	+1·08645	± ·0931
"	*45* to *70*	56·0605	6·1168	1·7563	·3877	·98178 ± ·00029	+ ·21675	± ·1450
"	*70* and over	76·4400	6·2166	1·4935	·2205	− ·85910 ± ·00824	− ·12683	—

* "Data for the Problem of Evolution in Man. V. On the Correlation between the Duration of Life and the Number of Offspring." Beeton, Yule and Pearson, *R. S. Proc.* Vol. LXVII. pp. 161—179.

† Unit of Standard Deviation of Age at Death is 5 years.

TABLE IV.

Table of Regression Formulæ.

x=Duration of Life. y=Size of Family.

(1) For lives of Fathers under 60 years of age

$$y = -2{\cdot}6570 + {\cdot}15391x.$$

(2) For lives of Fathers, 60 years of age and over

$$y = 5{\cdot}39489 + {\cdot}01328x.$$

(3) For lives of Mothers under 45 years of age

$$y = -3{\cdot}71792 + {\cdot}21729x.$$

(4) For lives of Mothers, 45 and under 70 years of age

$$y = 3{\cdot}6866 + {\cdot}04335x.$$

(5) For lives of Mothers, 70 years of age and over

$$y = 8{\cdot}1546 - {\cdot}02537x.$$

Note:—Origin at birth, and unit of x=1 year.

These lines, which are shown on the diagrams by broken lines, must be regarded as admirable representations of the statistics in all the series. Cubical parabolas were also computed for both sexes, but in neither case can it be said that the curves nearly as accurately represent the statistics as the regression straight lines. Adopting Professor Pearson's method* (mid-ordinate formula) the following equations for parabolas of the third order were obtained:

$$\text{For Males} \quad y = 6{\cdot}13673 + {\cdot}34808x - {\cdot}04913x^2 - {\cdot}00078x^3,$$

$$\text{„ Females} \quad y = 6{\cdot}29315 + {\cdot}15595x - {\cdot}04839x^2 + {\cdot}00199x^3,$$

the origin in the male curve being at 62·5 years, and in the female 60 years. The theoretical maximum fecundity in males was found to be at 78·925 years, and that in females at 69·075 years. Referring to the regression straight lines it is seen that in the case of males the coefficient of correlation in both series is positive; in the females in the first two periods it is positive, but in the period beyond 70 years of age it is negative. In all cases the coefficient is high and the probable error is very small. Thus it is seen that in man increased longevity is associated with increased fertility, whilst in women this association only exists up to about age 70, after which increased longevity is associated with diminished fertility, thus confirming Professor Pearson's suspicion that extreme fertility is not in all cases favourable to a maximum longevity. In man up to age 55—60, as might of course have been suspected, the association is well defined, but beyond that period it is not so pronounced. The association during the first period requires no explanation, being so intimately connected with the first or child-bearing period of women. But in the second period the connection between longevity and fertility is possibly more apparent than real. Many men who become widowers during that period contract subsequent marriage with

* *Biometrika*, Vol. II. Part I. pp. 16 and 17.

young women and thus increase their issue, whilst other married men, equal or superior in physique, whose wives still living survive the reproductive period, are necessarily unable to do so*. I have shown elsewhere† that the reproductive period in man, though diminishing in intensity from age 30—35, is practically not exhausted till about 75—80. Too much weight must not therefore be attached to this apparent association of extreme fertility and extreme longevity in man. There may be such association, but it is impossible to determine it from the available statistics. With females that increase of life should connote increased fertility during the period up to age 45 is too obvious to need comment; beyond that period it would seem that at all events so far as New South Wales is concerned, Weismann's views are supported—married women dying at ages 65—70 have had more offspring than either those who die younger or those who die older. Certainly extreme fertility is not associated with extreme longevity in females, and this phenomenon is probably brought about by the physical strain in bearing and the mental strain in rearing large families. Now the New South Wales statistics admit of this subject being analysed in another and perhaps in a more satisfactory manner. For the five years 1898 to 1902, the "Vital Statistics" contain tables showing the ages at death of married women (by single years), together with the number of children born to each woman. From these we are therefore enabled to ascertain the average duration of life of mothers of each sized family, and from any assigned period of life. Here it may be stated that throughout this investigation the term "mother" will be used instead of "married woman," notwithstanding that it may be regarded as an Hibernicism to speak of mothers of no children. From this data then I have determined the average duration of life of mothers beyond 46 years of age by survivors of age 45, and beyond ages 60 and 70 by survivors of those ages respectively. Obviously it would be prejudicial to the computed longevity of mothers of small families if the duration were determined as from any year prior to the termination of the reproductive period. That this period in New South Wales practically terminates with the close of the forty-fifth year is evident from the fact that of 335,642 confinements of married and single women which took place during the nine years 1893 to 1900 and 1902 (the tabulation in 1901 did not permit of the numbers of births to women aged 46 and upwards being obtained), I find that only 1013 or about three per 1000 were due to women who had attained the age of 46, and of these it may be of interest to mention that seventeen are recorded as due to women aged 50, six to women of 52, two to women of 53, one to a woman of 54, three to a woman of 55 (one of these was unmarried), and one each to women of 56 and 58 years of age respectively! In Table V. will be found the ages at death and number of offspring of the 10,276 women who died in New South Wales during the five years 1898 to 1902, and who had passed the reproductive period.

[* It seems desirable, as in the paper referred to on p. 235, to consider only single marriages, and not the children of two wives. Ed.]

† *Biometrika*, Vol. I. Part I. p. 34.

TABLE V.

Ages at Death of Married Women and Number of Offspring. Experience of New South Wales, 1898—1902.

Ages at Death	Number of Children in Family																							Total Deaths of Mothers
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*24*	*30*	
46	19	15	12	21	18	17	28	21	18	17	11	10	7	1	2	—	—	—	—	—	—	—	—	—	—	217
47	23	14	12	19	11	14	11	13	14	16	15	9	8	5	2	1	—	1	—	1	—	—	—	—	—	189
48	24	8	14	13	22	23	19	24	33	17	12	10	6	2	4	1	—	—	—	—	—	—	—	—	—	232
49	11	12	6	13	10	15	12	19	12	18	13	7	6	2	2	1	1	—	—	—	—	—	—	—	—	160
50	22	16	17	18	18	15	18	17	10	13	14	9	3	3	2	1	—	—	—	—	—	—	—	—	—	196
51	24	12	14	11	15	11	18	9	13	13	14	8	10	6	2	1	2	1	—	—	—	—	—	—	—	184
52	24	10	12	10	12	17	13	21	22	17	14	11	7	5	1	2	1	1	—	—	—	—	—	—	—	200
53	21	14	17	8	11	13	17	18	17	16	10	18	11	3	1	1	—	—	—	—	—	—	—	—	—	196
54	18	15	7	9	20	13	15	13	19	14	13	8	10	3	3	1	—	—	—	—	—	—	—	—	—	181
55	31	10	8	12	14	17	17	21	22	19	11	14	8	3	—	1	1	—	—	—	—	—	—	—	—	209
56	34	12	16	16	10	13	10	26	24	26	18	19	15	3	7	—	1	1	—	—	—	—	—	—	—	251
57	15	9	9	19	17	17	18	25	21	11	15	14	10	8	2	—	—	—	—	—	—	1	—	—	—	211
58	28	17	14	13	12	15	17	22	21	24	21	15	6	4	1	2	—	—	—	—	—	—	—	—	—	232
59	22	7	11	12	13	17	21	24	14	23	20	12	7	7	1	—	2	—	—	—	—	—	—	—	—	213
60	50	17	18	20	25	17	18	23	28	15	29	19	3	10	6	2	4	—	1	—	—	—	—	—	—	305
61	24	11	11	8	7	13	16	16	21	13	20	10	13	9	3	1	1	1	—	—	—	—	—	—	—	198
62	25	15	15	20	6	24	26	26	26	20	23	20	14	2	4	2	1	1	1	—	—	—	—	—	—	271
63	25	9	13	13	14	21	24	31	33	18	30	15	15	8	5	4	1	1	—	—	—	—	—	—	—	280
64	24	19	12	9	19	25	29	35	24	27	28	18	7	7	2	5	—	—	—	—	—	—	—	—	—	290
65	41	18	12	15	25	27	22	23	30	18	23	12	14	7	3	2	3	1	—	—	—	—	—	—	—	296
66	33	13	13	20	15	22	16	33	25	39	28	14	15	9	2	1	3	—	—	—	—	—	—	—	—	301
67	32	12	14	20	23	24	16	30	31	31	30	20	12	7	9	2	3	—	—	—	—	—	—	—	—	316
68	30	8	9	18	21	20	23	34	27	28	24	23	21	11	5	2	1	1	—	—	—	—	—	—	—	306
69	31	5	14	15	12	18	21	27	24	33	28	15	12	5	3	1	1	—	—	—	—	—	—	—	—	265
70	46	13	26	15	24	28	32	33	28	28	26	15	9	11	2	1	—	—	—	—	—	—	—	—	—	337
71	19	15	17	15	23	20	9	27	17	20	15	16	13	7	1	2	1	—	—	—	1	—	—	—	—	238
72	31	13	20	25	18	27	26	23	24	20	16	18	14	6	4	5	1	1	—	—	—	—	—	—	—	292
73	28	22	12	23	17	19	29	30	24	18	5	22	9	9	5	1	1	—	—	—	1	—	—	—	—	275
74	28	13	11	14	20	20	25	30	27	15	27	15	7	6	4	4	—	1	—	—	—	—	1	—	—	268
75	31	17	10	20	21	26	36	21	17	19	24	15	14	8	4	1	1	—	—	—	—	—	—	—	—	285
76	24	7	12	10	15	24	17	20	29	25	16	13	12	5	5	—	1	—	—	—	—	—	—	—	—	235
77	16	10	17	16	15	15	22	22	25	20	14	15	12	5	2	1	1	—	—	—	—	—	—	—	—	228
78	28	8	18	20	15	27	27	29	23	17	25	21	12	7	2	2	1	—	—	—	—	—	—	—	—	282
79	36	13	16	13	20	16	20	22	18	14	20	9	15	3	2	—	1	—	—	—	—	—	—	—	1	239
80	32	17	25	21	13	26	23	28	35	21	17	15	11	2	3	1	1	—	—	—	—	—	—	—	—	291
81	22	8	5	9	17	10	13	16	21	21	15	8	8	4	3	1	2	—	—	—	—	—	—	—	—	183
82	19	11	15	15	17	18	13	21	21	17	15	9	6	4	4	—	1	—	—	1	—	—	—	—	—	207
83	24	10	12	10	12	14	14	14	15	13	14	9	8	4	3	2	—	—	1	—	—	—	—	—	—	179
84	18	12	13	12	13	20	18	20	31	12	13	14	6	4	5	1	—	—	1	—	—	—	—	—	—	213
85	13	9	5	5	14	11	14	8	14	15	14	6	4	3	2	—	—	—	1	—	—	—	—	—	—	138
86	14	5	7	7	8	10	11	13	9	15	9	2	6	2	1	—	—	1	—	—	—	—	—	—	—	120
87	13	7	8	5	10	10	12	8	12	10	11	2	5	—	2	—	—	—	—	—	—	—	—	—	—	115
88	9	5	9	8	11	10	9	10	11	6	7	2	2	1	—	1	—	—	—	—	—	—	—	—	—	101
89	7	7	3	4	7	3	10	4	8	9	4	1	1	2	—	—	—	—	—	—	—	—	—	—	—	70
90	8	4	5	8	6	5	9	9	7	8	5	3	1	1	—	—	—	—	—	—	—	—	—	—	—	79
91	2	—	2	3	2	3	4	1	4	3	—	1	1	—	—	—	1	—	—	—	—	—	—	—	—	27
92	3	2	2	2	4	4	4	8	2	3	2	3	3	—	—	—	—	—	—	—	—	—	—	1	—	43
93	2	3	2	4	2	2	3	1	2	4	2	2	1	—	1	—	—	—	—	—	—	—	—	—	—	31
94	—	—	1	4	2	5	2	1	3	2	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	24
95	1	—	3	2	1	—	2	—	2	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	14
96	2	3	1	—	—	7	3	3	—	2	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	24
97	1	—	1	—	—	3	—	—	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
98	1	—	—	1	2	1	2	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	8
99	—	—	1	—	1	1	1	1	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	8
100	—	—	2	—	—	—	—	—	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
101	—	—	—	1	2	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
102	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
103	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
104	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
105	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	1110	533	581	644	702	813	855	976	963	847	786	568	422	226	129	57	39	12	5	2	2	1	1	1	1	10,276

Although the last column shows the usual aggregations at the decennial years of age, it was not found practicable to graduate or adjust the numbers in consequence of the number of offspring also being involved. This will not however vitiate the comparison between the mothers of the different sized families, as the error is common to all. The average duration of life was determined by means of the single years and a half-year added to each result to allow for the life lived in the year of death on the assumption of the equal distribution of deaths throughout that year. The statistics relating to families of more than seventeen children were not used, being altogether too small to furnish reliable results. The average duration of life beyond age 45 of the mothers of various sized families who had survived their forty-sixth year, and of those beyond 60 and 75 by survivors of those ages, is given in Table VI.

TABLE VI.

Mean Duration of Life of Mothers.

Number of children in Family	After 45 years of age		After 60 years of age		After 75 years of age	
	Number of Deaths	Average Life (years)	Number of Deaths	Average Life (years)	Number of Deaths	Average Life (years)
0	1110	22·63	794	13·33	327	6·90
1	533	22·29	362	14·15	159	7·63
2	581	23·41	412	14·76	195	7·64
3	644	22·82	450	14·36	200	7·46
4	702	23·39	499	14·75	230	7·73
5	813	23·68	596	14·42	271	7·51
6	855	23·61	621	14·53	289	7·33
7	976	22·91	703	13·54	282	7·18
8	963	23·41	703	14·06	314	7·38
9	847	23·14	603	14·10	260	7·57
10	786	22·96	585	12·91	233	6·95
11	568	22·26	404	12·75	152	6·15
12	422	22·94	308	13·29	130	6·19
13	226	22·68	171	12·06	57	6·22
14	129	23·62	99	13·42	41	7·16
15	57	21·94	45	10·72	10	6·20
16	39	23·24	31	10·60	10	6·20
17	12	18·58	8	9·38	—	—

Guided approximately by the statistics, each of these series was broken up into two sections—from 0 to 8 children and from 9 to 17 in the first two, and from 0 to 8 and 9 to 16 children in the last series. The correlation and the regression straight lines were then determined for the six sections with the results set forth in Tables VII. and VIII.

Although the correlation cannot be regarded as high in any of the series there are some significant facts in connection with the results—first, the correlation in each of the series 0 to 8 children is positive, and from 9 children upwards it is negative; that is, that fertility up to about 8 children is apparently

TABLE VII.

General Results.

Period of Life	Size of Family	Mean Duration of Life (years)	Mean Size of Family	Standard Deviation		Correlation. Duration of Life and Fertility	Regression
				Duration of Life (Unit =1 year)	Size of Family		
After 45 years	0 to 8 children	23·1369	4·2031	12·2551	2·7189	+·02144 ±·00796	+·09666
„ „	9 to 17 „	22·8396	10·7654	11·4486	1·6648	−·01285 ±·01214	−·08834
After 60 years	0 to 8 „	14·1321	4·2379	8·7413	2·7189	+·009205 ±·00941	+·02959
„ „	9 to 17 „	13·1207	10·7289	8·3649	1·6825	−·07192 ±·01413	−·35757
After 75 years	0 to 8 „	7·3796	4·2541	5·3257	2·6810	+·00710 ±·01420	+·01410
„ „	9 to 16 „	6·8203	10·6685	5·0405	1·5830	−·08687 ±·02240	−·27662

TABLE VIII.

Regression Formulæ.

y = Duration of Life; x = Size of Family (Origin at 0 children).

(1) For lives of Mothers of 0 to 8 children, after 45 years of age, $y = 22{\cdot}7306 + {\cdot}09666x$.
(2) „ „ 9 to 17 „ „ „ $y = 23{\cdot}7906 - {\cdot}08834x$.
(3) „ „ 0 to 8 „ 60 „ $y = 14{\cdot}0067 + {\cdot}02959x$.
(4) „ „ 9 to 17 „ „ „ $y = 16{\cdot}9808 - {\cdot}35757x$.
(5) „ „ 0 to 8 „ 75 „ $y = 7{\cdot}3196 + {\cdot}01410x$.
(6) „ „ 9 to 16 „ „ „ $y = 9{\cdot}7714 - {\cdot}27662x$.

favourable to longevity, whilst unfavourable beyond that number; secondly, the mean family of the 0 to 8 children groups increases with each period, whilst the means of the 9 to 17 children groups as regularly diminish; and thirdly, that the mean duration of lives of mothers of the 0 to 8 children groups is consistently greater than that of the mothers of larger families. These statistics have also been fitted with cubical parabolas (true ordinate formula) with the following results:

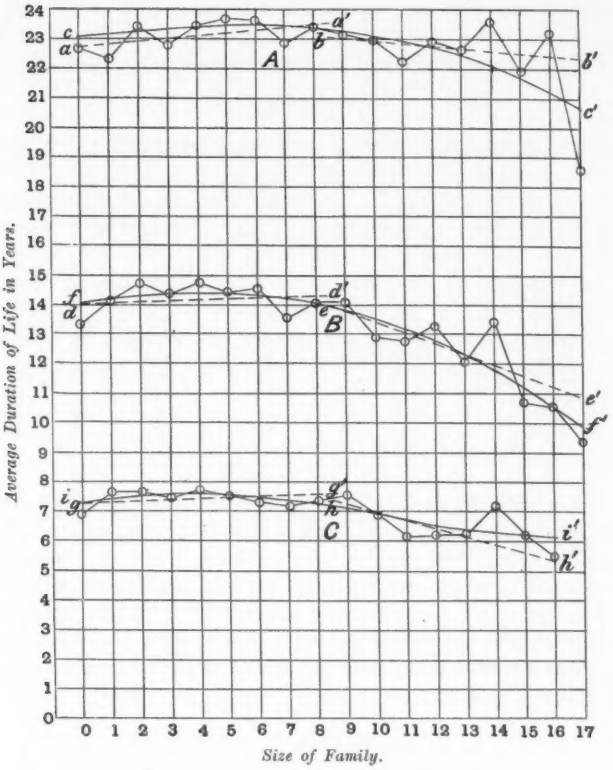
For life after age 45, $y = 23{\cdot}32576 - {\cdot}08538x - {\cdot}01864x^2 - {\cdot}00071x^3$,
„ „ „ „ 60, $y = 13{\cdot}94679 - {\cdot}21964x - {\cdot}02765x^2 - {\cdot}00032x^3$,
„ „ „ „ 75, $y = 7{\cdot}20525 - {\cdot}15409x - {\cdot}00816x^2 + {\cdot}00135x^3$.

The origin of the first two curves is at 9 children, and of the latter at 8·5 children. The maxima were found to be at 5·790, 4·208, and 3·527 children respectively. The statistics, regression straight lines and curves are plotted on Fig. 3.

Although as has been previously remarked the correlation is not high in any of the series, yet in the case of life beyond 60 and beyond 75 for mothers of more than eight children, it is sensibly negative within the limits of probable error, and this is sufficient to justify the conclusion that even in the case of survivors attaining those ages, their future expectation of life has been prejudiced by every additional child that had been born to them. But if excessive fertility

Fig. 3. Number of Offspring and Average Duration of Life of Mothers.

A, Mean Lifetime after 45 Years of Age, by Survivors of Age 46.
B, " " 60 " " " 60.
C, " " 75 " " " 75.



Equations to Regression Straight Lines:

$$\left.\begin{aligned} aa',\ y &= 22{\cdot}7306 + {\cdot}09666x \\ bb',\ y &= 23{\cdot}7906 - {\cdot}08834x \\ dd',\ y &= 14{\cdot}0067 + {\cdot}02959x \\ ee',\ y &= 16{\cdot}9808 - {\cdot}35757x \\ gg',\ y &= 7{\cdot}3196 + {\cdot}01410x \\ hh',\ y &= 9{\cdot}7714 - {\cdot}27662x \end{aligned}\right\} \text{Origin at 0 children. Unit of } x=1 \text{ child.}$$

Equations to Cubical Parabolas:

$$cc',\ y = 23{\cdot}32576 - {\cdot}08538x - {\cdot}01864x^2 - {\cdot}00071x^3 \left\{\begin{matrix}\text{Origin at } 8{\cdot}5 \text{ children.} \\ \text{Unit of } x=1 \text{ child.}\end{matrix}\right.$$

$$ff',\ y = 13{\cdot}94679 - {\cdot}21964x - {\cdot}02765x^2 - {\cdot}00032x^3 \left\{\begin{matrix}\text{Origin at } 8{\cdot}5 \text{ children.} \\ \text{Unit of } x=1 \text{ child.}\end{matrix}\right.$$

$$ii',\ y = 7{\cdot}20525 - {\cdot}15409x - {\cdot}00816x^2 + {\cdot}00135x^3 \left\{\begin{matrix}\text{Origin at } 8 \text{ children.} \\ \text{Unit of } x=1 \text{ child.}\end{matrix}\right.$$

be prejudicial to longevity at all, it must be only those of exceptional physique who have attained the ages of 60 and 75 years, and the more satisfactory method of determining the correlation between fertility and longevity would be to take into account the deferred expectation beyond those ages by survivors of age 46. (It may be remarked that the expectation here referred to is not the actuarial expectation of life—but of that hereafter.) This expectation has been calculated for mothers of each sized family, and the results are shown in Table IX.

TABLE IX.

Average Duration of Life by Mothers Surviving Age 46.

Number of children in Family	Duration beyond 60 years of age	Duration beyond 75 years of age	Number of children in Family	Duration beyond 60 years of age	Duration beyond 75 years of age
0	9·53 years	2·03 years	*9*	10·03 years	2·32 years
1	9·61 „	2·28 „	*10*	9·61 „	2·06 „
2	10·47 „	2·56 „	*11*	9·07 „	1·65 „
3	10·04 „	2·32 „	*12*	9·70 „	1·91 „
4	10·49 „	2·53 „	*13*	9·13 „	1·57 „
5	10·57 „	2·50 „	*14*	10·30 „	2·28 „
6	10·56 „	2·48 „	*15*	8·46 „	1·09 „
7	9·75 „	2·08 „	*16*	8·43 „	1·41 „
8	10·26 „	2·41 „	*17*	6·25 „	0·96 „

These results were obtained by dividing the total life beyond 60 and beyond 75 years of age by the number of mothers who had survived their forty-sixth year and adding a half year for the year of death. Taking advantage of the knowledge derived from the parabolas of the preceding investigation that the maximum duration of life was about six children, these results were broken up at that number and the correlation and regression straight lines were determined at each period as from 0 to 6 children and from 6 to 17 children, with the results set forth in Tables X. and XI.

TABLE X.

General Results.

Period of Life	Size of Family	Mean Duration of Life (years)	Mean Size of Family	Standard Deviation		Correlation. Duration of Life and Fertility	Regression
				Duration of Life (Unit = 1 year)	Size of Family		
After 60 years	0 to 6 children	10·1636	2·9840	·44058	2·1561	+ ·89004 ± ·00194	+ ·18187
„ „	6 to 17 „	9·8640	8·9946	·24559	2·2957	− ·68479 ± ·00467	− ·14782
After 45 years	0 to 6 „	2·3647	2·9840	·19396	2·1561	+ ·64963 ± ·00539	+ ·05844
„ „	6 to 17 „	2·1366	8·9946	·30305	2·2957	− ·74144 ± ·00396	− ·09788

TABLE XI.

Table of Regression Formulæ.

y = Duration of Life; x = Size of Family.

(1)	Life after	60	years of age,	Mothers of	0 to 6	children,	$y = 9{\cdot}6209 + {\cdot}18187x$
(2)	„	„	„	„	6 to 17	„	$y = 11{\cdot}1936 - {\cdot}14782x$
(3)	„	75	„	„	0 to 6	„	$y = 2{\cdot}1903 + {\cdot}05844x$
(4)	„	„	„	„	6 to 17	„	$y = 3{\cdot}0170 - {\cdot}09788x.$

Origin at 0 children. Unit of $x = 1$ child.

Cubic parabolas were also determined for these two series, and were found to be—the origin of both curves being at 8·5 children—

For life after 60 years of age

$$y = 10{\cdot}465864 - {\cdot}099753\,x - {\cdot}033096\,x^2 - {\cdot}000352\,x^3.$$

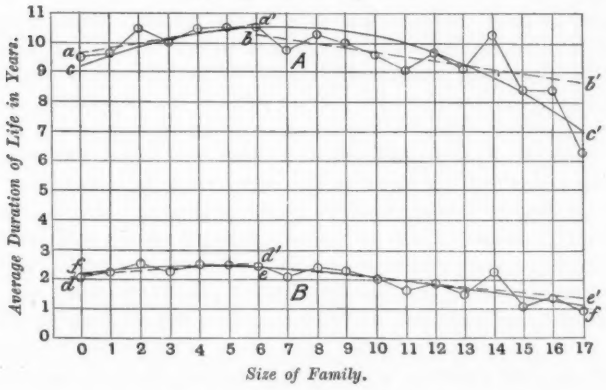
For life after 75 years of age

$$y = 2{\cdot}245851 - {\cdot}089609\,x - {\cdot}007900\,x^2 + {\cdot}000422\,x^3.$$

The theoretical maxima were found to be at 6·955 and 5·268 children respectively. The statistics, regression straight lines and curves are plotted on Fig. 4.

FIG. 4. Number of Offspring and Average Duration of Life of Mothers surviving Age 46.

A, Beyond 60 years of Age.
B, „ 75 „



Equations to Regression Straight Lines:

aa', $y = 9{\cdot}6209 + {\cdot}18187x$
bb', $y = 11{\cdot}1936 - {\cdot}14782x$
dd', $y = 2{\cdot}1903 + {\cdot}05844x$
ee', $y = 3{\cdot}0170 - {\cdot}09788x$

} Origin at 0 children. Unit of $x = 1$ child.

Equations to Cubic Parabolas:

cc', $y = 10{\cdot}465864 - {\cdot}099753x - {\cdot}033096x^2 - {\cdot}000352x^3$
ff', $y = 2{\cdot}245851 - {\cdot}089609x - {\cdot}007900x^2 + {\cdot}000422x^3$

} Origin at 8·5 children. Unit of $x = 1$ child.

In these deferred expectations the correlation is very high and within very small limits of error, and again it is positive in both series for mothers of small families and negative for mothers of large families. We may therefore safely conclude that the advantage in longevity is distinctly in favour of mothers of moderate sized families of about six children, and that in New South Wales at all events, maximum fertility is extremely unfavourable to extreme longevity. I think the reasons are sufficiently obvious—and what applies to New South Wales must apply with equal force elsewhere—the incessant strain upon the physique of women who bear large families during the periods of gestation, parturition and lactation must be very prejudicial to longevity, whilst the mental strain involved in the rearing of such families cannot be regarded as other than detrimental. But we are also presented with the fact that childless women and mothers of extremely small families have shorter expectation of life than mothers of moderate sized families. This I consider must arise partly from the inferior physique of unprolific women generally, and partly—perhaps chiefly—in consequence of the prejudicial effects of the efforts to prevent and to limit families. But whatever may be the explanation, the shortened life thus occasioned is by no means equal to that occasioned by unrestrained excessive fertility. In New South Wales there is but little Malthusian restraint when compared with the old countries of the world—notwithstanding that the Government of that State, alarmed at the decline in the birthrate, appointed a Royal Commission to investigate the causes operating to produce such decline. In its Report, relying upon medical evidence and on statistics which I think were wrongly interpreted, it declared that there had been a serious decline and that this was chiefly to be attributed to artificial restraint. That there has been a decline in the birthrate, whether measured as per 1000 of the population, or per 1000 married women of the reproductive period, or in age groups within that period, is obvious from the figures produced in evidence, but it was entirely overlooked when dealing with the latter method—the only correct one—that there had been a considerable change in the physique of the population. In the earlier days the community consisted chiefly of specially selected immigrants drawn for the most part from the most prolific classes of the British community, viz. the agricultural and artisan. Now that immigration has practically ceased, the birthrate has declined from an abnormal condition due to a differentiated community to that of a more normal population. That there is restriction to some extent amongst certain classes of the community is doubtless true—although it is equally probable that this has always been the case—but, as we shall subsequently see in another part of this paper*, the extent of this practice is very limited.

It was previously remarked that the expectation or mean duration of life used throughout the investigation should not be mistaken for the actuarial expectation of life at the ages referred to. Thus according to the English Registrar-General's life table†, the expectation of females aged 46 is 23·34 years,

* See "Distribution of Fertility," p. 247.

† Supplement to *Fifty-fifth Annual Report*, Part I. pp. xviii. and xix.

so that the expectation beyond 45 of those surviving age 46 is 24·34 years; the expectation at age 60 is 14·10 and at age 75, 6·68 years*. The complete expectations at similar ages computed by the present writer from the New South Wales census experience of 1891 are 26·17, 14·37, and 7·02 years respectively. The average durations of life determined from the experience of married women in the present investigations are 23·05, 13·82, and 7·22 years respectively. It is seen that at the first two ages the averages for married women are less than the expectations according to both the English and New South Wales life tables for the general population of females, although in excess at the later period. At first sight it might appear that the married condition was somewhat unfavourable to longevity; but this is not so. We are able to compare, not the actuarial expectation, but the mean duration of life of married and celibate men and women upon exactly the same basis. The ages at death of each sex are tabulated in quinquennial age groups, and by subtracting the married, the numbers of bachelors and spinsters have been obtained as shown in Table XII.

TABLE XII.

Number of Deaths of each Sex, Single and Married.

Age at Death	Deaths of Females (1898 to 1902)			Deaths of Males (1901 and 1902)		
	All Females	Married	Single	All Males	Married	Single
45 and under *50*	1181	1050	131	939	616	323
50 „ *55*	1093	972	121	851	588	263
55 „ *60*	1218	1126	92	931	652	279
60 „ *65*	1484	1362	122	1083	750	333
65 „ *70*	1609	1491	118	1382	937	445
70 „ *75*	1513	1423	90	1276	871	405
75 „ *80*	1356	1281	75	830	606	224
80 „ *85*	1132	1080	52	589	433	156
85 „ *90*	579	551	28	310	271	39
90 „ *95*	219	206	13	129	98	31
95 „ *100*	69	65	4	41	33	8
100 „ *105*	14	14	—	13	9	4
Totals ...	11,467	10,621	846	8374	5864	2510

Note:—The numbers given in the columns headed "Married" do not agree with those given in Tables I. and II., as in this table are included those mothers whose ages at death were known but the numbers of whose offspring were unknown. The latter were of course excluded from Tables I. and II.

From the data in this table the average durations of married and celibate life were determined, the central age of each age group being adopted as the mean age of that group. The results for females were as follows†:

* I cannot tell from the table referred to whether the expectations given are complete or curtate.

† It will be noticed that the averages determined by the adoption of central ages of a quinquennial group differ but slightly from those determined by the adoption of single years.

Average duration of life after 45 years of age:

Married Women	22·60 years.
Single do.	18·98 „

Average duration of life after 60 years of age:

Married Women	13·91 years.
Single do.	12·23 „

Average duration of life after 75 years of age:

Married Women	7·40 years.
Single do.	7·24 „

At each period there is exhibited a sensible superiority of the married over the unmarried state, and this difference being undoubtedly real, the natural inference is that the exercise of the functions incidental to the married state is conducive to longevity—at all events after the reproductive period. It can hardly be urged that an unmarried woman who survives the age of 45 suffers generally from any constitutional infirmity which, from motives of prudence or otherwise, deterred her from contracting marriage or which physically incapacitated her. The greatest enemy of mankind—phthisis—has at that age practically exhausted itself. A life at that age may be regarded as a selected life, but more and more undoubtedly so at ages 60 and 75. For males the results were*:

Average duration of life after 45 years of age:

Married Men	21·59 years.
Single do.	19·98 „

Average duration of life after 60 years of age:

Married Men	13·07 years.
Single do.	11·68 „

Average duration of life after 75 years of age:

Married Men	7·49 years.
Single do.	6·60 „

Again we find the differences consistently in favour of the married state—not so pronounced at the first two periods as amongst females, but somewhat more so at the latest. In view of the results obtained for both sexes, we are warranted in concluding that marriage, at all events so far as longevity is concerned, is not a failure.

It may be remarked that the material we have been dealing with may be said to be homogeneous—the deaths are those of persons almost entirely of British stock or origin. Of the 11,396 deaths of females which occurred during the years 1898 to 1902 aged 45 and upwards, 2572 or 22·5 per cent. were of Australasian birth; 8406 or 73·8 per cent. were born in the United Kingdom, whilst only 418 or 3·7

* It will be noticed that the averages determined by the adoption of central ages of a quinquennial group differ but slightly from those determined by the adoption of single years.

per cent. were born elsewhere. Of the 6457 who died at age 65 or over these figures are 665 or 10·3 per cent., 5545 or 85·9 per cent., and 247 or 3·8 per cent. respectively.

From the results of this section of the investigation we are amply justified in concluding:

(*a*) That in females extreme fecundity is unfavourable to extreme longevity.

(*b*) That mothers of moderate sized families of about six live on the average longer than those with smaller or larger families.

(*c*) That although in males increased fertility is apparently associated with increased longevity throughout the whole of life, such association in part at least is due to second and subsequent marriages.

(*d*) That the married of both sexes have a greater expectation of life than the unmarried, and that this superiority is not all due to physical infirmities which prevented marriage.

B. *Distribution of Fertility.*

Professor Pearson, in his essay "Reproductive Selection*," has dealt with this subject rather exhaustively, using the data derived from certain sections of the Anglo-Saxon community and from the upper and middle classes of the city of Copenhagen tabulated by Messrs Rubin and Westergaard. The statistics now to be presented are for the whole state of New South Wales—urban and rural—and will exhibit the distribution of fertility under varying conditions of ages of women and durations of marriage. Taking first of all the women used in the last investigation, i.e. those who died beyond the reproductive period irrespective of the age at marriage or of the duration of marriage, the distribution is as shown in Table XIII.

From this table we see that nearly 11 per cent. of the women were sterile, and it would appear from Fig. 5 (broken lines) that this is unduly high. The excess might at first sight be attributed to absolute prevention of family, but as these statistics relate to married women dying beyond the reproductive period, they comprise widows—whether widowed before or after the reproductive period—and of women who may have married late in that period and whose fertility had but little opportunity to display itself†, and of women who may have married beyond that critical period and consequently no opportunity whatever. Therefore although this percentage of barren women agrees fairly closely with that obtained by Rubin and Westergaard for marriages of duration of 15 years and upwards with both husband and wife alive, viz. 12·87, this ratio is distinctly too high for New South Wales. For marriages under the last-mentioned conditions, as we shall subsequently see, the barren marriages are but 5·563 per cent., which

* *Chances of Death*, pp. 63 *et seq.*

† Vide *Biometrika*, Part I. Vol. I. p. 33, see curve showing initial fertility in deferred marriages.

TABLE XIII.

Distribution of Fertility. Women dying beyond the Reproductive Period. Experience of New South Wales.

Size of Family	Deaths of Married Women	Per Cent. of Total	Sum of Percentages	Number of Children	Per Cent. of Total	Sum of Percentages
0	1110	10·802	10·802	—	—	—
1	533	5·187	15·989	533	·837	·837
2	581	5·654	21·643	1162	1·826	2·663
3	644	6·267	27·910	1932	3·035	5·698
4	702	6·831	34·741	2808	4·411	10·109
5	813	7·912	42·653	4065	6·230	16·339
6	855	8·320	50·973	5130	8·216	24·555
7	976	9·498	60·471	6832	10·733	35·288
8	963	9·371	69·842	7704	12·103	47·391
9	847	8·243	78·085	7623	11·976	59·367
10	786	7·649	85·734	7860	12·348	71·715
11	568	5·527	91·261	6248	9·816	81·531
12	422	4·107	95·368	5064	7·956	89·487
13	226	2·199	97·567	2938	4·615	94·102
14	129	1·255	98·822	1806	2·838	96·940
15	57	·555	99·377	855	1·343	98·283
16	39	·380	99·757	624	·980	99·263
17	12	·116	99·873	204	·320	99·583
18	5	·049	99·922	90	·142	99·725
19	2	·020	99·942	38	·060	99·785
20	2	·019	99·961	40	·063	99·848
21	1	·010	99·971	21	·033	99·881
22	1	·010	99·981	22	·034	99·915
24	1	·010	99·991	24	·038	99·953
30	1	·009	100·000	30	·047	100·000
Totals	10276	100·000	—	63653	100·000	—
Median	5·8831			8·2179		
	Mean Family = 6·1943 children					

agrees remarkably close with that computed by Professor Pearson for the Danish statistics, viz. 5·717. This is of course for all women at whatever age married, but whose marriage had existed 15 years or more at the time of the New South Wales census of 1901. From the returns of that census I have been enabled to compile not only the durations of marriage but the ages of the women at the time of the census together with the number of childless women. The results are shown in Table XIV.

This table shows that under suitable conditions of age at marriage and duration of marriage, the naturally sterile women do not exceed 2½ per cent. of the total, and even of that small percentage some part is probably due to impotent husbands. We see also from this table that barrenness increases with the age at marriage,

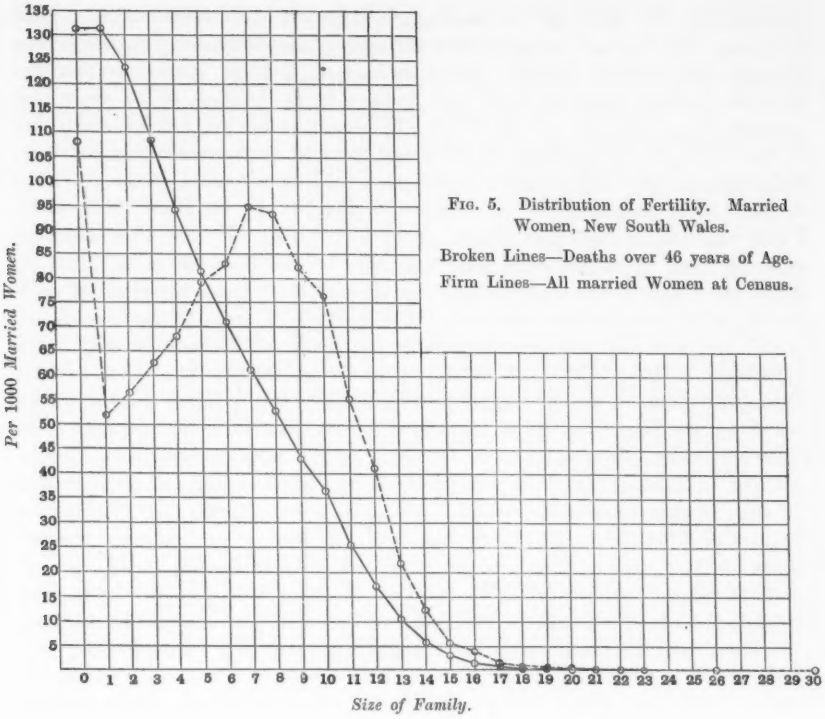


FIG. 5. Distribution of Fertility. Married Women, New South Wales.
Broken Lines—Deaths over 46 years of Age.
Firm Lines—All married Women at Census.

TABLE XIV.

Age of Wife at Census 1901	Duration of Marriage at Census 1901								
	10 Years and Over			15 Years and Over			20 Years and Over		
	Total Wives	Without Issue		Total Wives	Without Issue		Total Wives	Without Issue	
		Number	Per Cent.		Number	Per Cent.		Number	Per Cent.
20 and under *25*	6	1	16·67	—	—	—	—	—	—
25 „ *30*	3712	119	3·21	5	—	—	—	—	—
30 „ *35*	18000	726	4·03	3438	89	2·59	7	—	—
35 „ *40*	26520	1281	4·83	16458	536	3·26	2799	68	2·43
40 „ *45*	23904	1183	4·95	20681	740	3·58	12672	309	2·44
45 „ *50*	17169	1165	6·79	16137	878	5·44	13771	598	5·98
50 „ *55*	35294	3503	9·93	34338	2818	8·21	32848	2127	6·47
Totals ...	124605	7978	6·40	91057	5061	5·56	62097	3102	5·00

and that for the same age at marriage it decreases with its duration. Thus of women who married at age under 20, and whose duration of marriage was 10 years and upwards, 3·21 per cent. were barren; of those whose duration was 15 years and upwards, 2·59 per cent., whilst for a duration of 20 years and upwards the percentage was only 2·43. These differences are I think really due to first births occurring from 10 to 15 and from 15 to 20 years after marriage, rather than to increased restraint in the more modern marriages, for on reference to the "Vital Statistics" of New South Wales for the 10 years 1893—1902, I find that the average annual first births from 10 to 15 years after marriage were 24, from 15 to 20 years after marriage 5, and from 20 to 25 years ·5. When the number of marriages and the proportion of those marrying at such ages as to be able to bear children at the intervals referred to are considered, as also the ruptured marriages caused by the death of husband or wife, there is little or nothing left to be accounted for by restraint in the more modern marriages. It seems safe to assume then that the naturally sterile women cannot exceed 2½ per cent. So far as I am aware these statistics are the first from which an accurate determination of the value of this ratio can be made. Again referring to Table XIII., it is seen that the condition of things existing after the close of the reproductive period is that the average family is 6·19 children, the median 5·88, the mode 7, and that 28·36 per cent. of such women produce 50 per cent. of the children. If, however, the married women at all ages at death be taken, it is found that the average family is 5·31 children, the median 4·55, and that 25·61 per cent. of the women produce 50 per cent. of the children. This latter result—based on what I regard as the true method, being on deaths at all ages—is almost in exact accordance with Prof. Pearson's conclusion from the Anglo-Saxon and Danish statistics, viz. "that one quarter of the popula-

TABLE XV.

Distribution of Fertility. Duration of Marriage 15 *years and upwards. Experience of New South Wales Census* 1901.

Size of Family	Families		Size of Family	Families		Size of Family	Families	
	Number	Per Cent.		Number	Per Cent.		Number	Per Cent.
0	5074	5·563	*9*	8420	9·231	*18*	67	·073
1	3309	3·628	*10*	7260	7·959	*19*	32	·035
2	4079	4·472	*11*	5076	5·565	*20*	10	·011
3	5416	5·938	*12*	3550	3·892	*21*	9	·010
4	6909	7·575	*13*	2078	2·278	*22*	4	·004
5	8273	9·070	*14*	1171	1·284	*23*	3	·003
6	9551	10·471	*15*	591	·648	*26*	1	·001
7	10022	10·987	*16*	283	·310			
8	9907	10·861	*17*	119	·131	Totals	91214	100·000

tion produce one half of the next generation*." This additional corroboration affords strong evidence of its universal application.

Next taking the marriages, the duration of which is 15 years and upwards, the distribution of fertility is as shown in Table XV.

A comparison of these statistics with the similar statistics of Messrs Rubin and Westergaard's quoted by Prof. Pearson will show an immense superiority of fertility in favour of the New South Wales woman, but it must be borne in mind that the Danish statistics refer only to the professional and middle classes of the city of Copenhagen, and which necessarily exclude the most prolific sections (at least it is so in New South Wales, as will be subsequently shown in the section "Reproductive Selection") of the community, viz. the artisan and the agricultural and pastoral. Here we find 5·56 of the marriages are childless, and as this appears unduly high I have, in calculating the fertility curve, altered the value to 2 per cent., and the computed curve justified the alteration. The following were found to be the constants:

$\mu_2 =$	11·32501	Mean Family	= 6·95103
$\mu_3 =$	4·88272	Modal Family	= 5·68098
$\mu_4 =$	350·82375	Median Family	= 8·03020 (Observed)
$\beta_1 =$	·016414	Range	= 29·51447
$\beta_2 =$	2·735343	Standard Deviation	= 3·3653
$\epsilon =$	77·7792	y_0	= 109·9507

and the equation to the curve of fertility

$$y = 109{\cdot}9507 \left(1 + \frac{x}{12{\cdot}3500}\right)^{6{\cdot}6225} \left(1 - \frac{x}{17{\cdot}1645}\right)^{9{\cdot}2039}.$$

In this series it is found that 31·16 of the women produce one half of the children. The statistics and curve will be found plotted on Fig. 6.

On comparing the constants of this curve with those of the Danish or Anglo-Saxon curves, it is seen that we are dealing either with a naturally more prolific community or one in which there is little resort to Malthusian restraint; for although the artisan section of the community is not included in the former statistics, yet the mean family of the New South Wales woman married 15 years and upwards, viz. 6·951, is far in excess of the mean Danish artisan family for marriages of 25 years and upwards duration, viz. 5·26†. As the Scandinavian race is unquestionably a very prolific race, and the Australian mothers are, as shown in Section A, of essentially British stock—which, as Prof. Pearson remarks‡, is markedly less fertile than the Danish—it must be concluded that but little Malthusian restraint is practised in New South Wales: yet, as previously stated, the Government of that State deemed it necessary to appoint a Royal Commission to enquire into the causes of the so-called decline in the birthrate. That Com-

* *Chances of Death*, p. 92.

† *Loc. cit.* p. 98. ‡ *Loc. cit.* p. 89.

mission concluded "That there is a diminution of fecundity and fertility in recent years which is due to:

(*a*) Deliberate prevention of conception and destruction of embryonic life;

(*b*) Pathological causes consequent upon the means used and the practices involved therein;

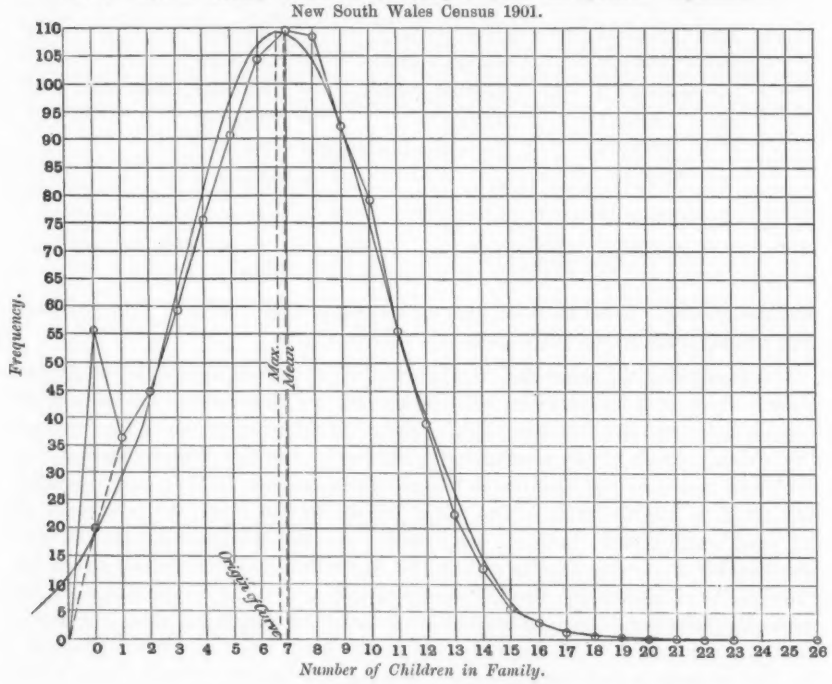
"and from the evidence of medical witnesses, police officers and others, we are led to the conclusion that:

(*a*) The practice of preventing conception by artificial means is common among all classes of the community and in all parts of the State;

(*b*) This practice has greatly increased during the last 15 years; and

(*c*) The extension of the deliberate and intentional avoidance of procreation by means used to prevent conception is a main factor in producing the decline in birthrate, which we have ascertained to have taken place in New South Wales since the year 1888."

FIG. 6. Curve of Fertility. Duration of Marriage 15 years and upwards. Experience of New South Wales Census 1901.



Equation to Curve $y = 109{\cdot}9507\left(1+\frac{x}{12{\cdot}3500}\right)^{6{\cdot}6225}\left(1-\frac{x}{17{\cdot}1645}\right)^{9{\cdot}2039}$.

Mean Family = 6·95103.

Modal „ = 6·68020.

It is not my intention to deal with this subject from the ethical standpoint, but a thorough statistical analysis compels me to disagree not only with the conclusions of the Commission but the unscientific reasons in support thereof. It is not necessary here to traverse the evidence given, but it will suffice to produce the actual facts as recorded at the census. It is clearly seen from the distribution of fertility as revealed in Table XV. that we are dealing with a naturally most fertile race. It may be reasonably argued that the distribution shown in that table refers only to marriages which were contracted at least fifteen years ago, and not to present marriages. But we can well judge of the effect of the alleged fifteen years' excessive restriction if we examine the women who were married from 15 to 20 years at the time of the census and who were then under 40 years of age. The results I find to be as follow :

Size of Family	Number of Families	Size of Family	Number of Families
0	553	*6*	2662
1	637	*7*	2570
2	948	*8*	2063
3	1402	*9*	1125
4	1967	*10*	539
5	2235	*11* and over*	369

Here it is seen that (allowing only 11 children to the 369 women who had 11 and over) the mean family is 5·61 children and only 3·24 per cent. who had so far proved sterile, and consequently these figures utterly refute the Commission's conclusions. In view of these figures, we can assert that there cannot have been much restraint practised in New South Wales during the last fifteen years. I wish to emphasise this point in order that it may be recognised that the curves of fertility for various durations of marriage in New South Wales are as nearly as possible those free from restraint. That restraint is practised by certain sections of the community I am well aware, as also that it always has been so in some form or other, but in view of the figures which have been produced and those to be produced, I think it is altogether erroneous to assert that the practice is common to all sections of the community and to all parts of the State.

Now let us ascertain the distribution for durations of marriage of less than 15 years. I have taken those women who were under 20 years of age at marriage, and have divided the duration of marriage into three periods—from 4 to 5 years, from 5 to 10 years, and from 10 to 15 years—the ages at the census being under 30 for the first two, and under 35 for the latter period. The statistics and percentages are shown in Table XVI.

* The returns do not show the distribution beyond 10 children.

TABLE XVI.

Duration of Marriage and Number of Offspring. Women's Age at Marriage under 25. Experience of New South Wales Census 1901.

Number of Offspring	Duration 4 to 5 Years		Duration 5 to 10 Years		Duration 10 to 15 Years	
	Married Women	Percentage	Married Women	Percentage	Married Women	Percentage
0	541	9·47	1094	6·05	757	4·14
1	1386	24·27	2230	12·33	1039	5·68
2	2731	47·81	4811	26·60	1874	10·25
3	954	16·70	5567	30·78	2931	16·03
4	96	1·68	3124	17·27	3603	19·71
5	4	·07	996	5·51	3715	20·32
6	—	—	226	1·25	2542	13·91
7	—	—	36	·20	1233	6·75
8	—	—	2	·01	422	2·31
9	—	—	—	—	118	·65
10	—	—	—	—	38	·21
11	—	—	—	—	7	·04
Totals	5712	100·00	18086	100·00	18279	100·00

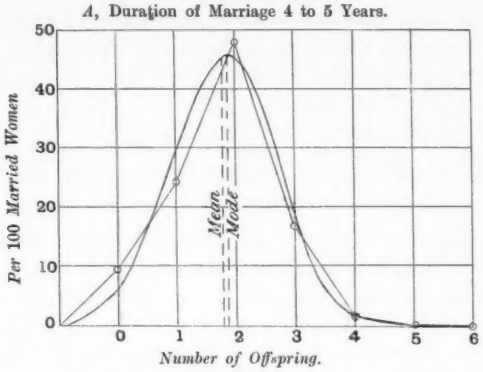
These percentages have been plotted on Fig. 7, and the fertility curves computed. The constants were :

	Duration of Marriage.		
	4 to 5 years	5 to 10 years	10 to 15 years
μ_2	·72604	1·6271	3·6399
μ_3	− ·11092	+ ·1283	− ·5179
μ_4	1·51477	7·7554	36·6377
β_1	·032146	·003821	·005562
β_2	2·873581	2·929381	2·765345
Range	10·0683	22·5456	18·2893
Mean Family	1·7706	2·6348	4·1229
Modal Family	1·8631	2·5932	4·2086
Standard Deviation	·8521	1·2756	1·9078
y_0	45·9164	30·9904	20·2118

It is worthy of notice that for marriages of an average duration of 4·5 years the mean family is 1·77, and that for an average duration of 7·5 years it is 2·63—the difference of 3 years' duration being responsible for ·86 child or ·288 annually. The difference in mean families for marriages of $7\frac{1}{2}$ and $12\frac{1}{2}$ years' mean duration is 1·488 child or ·298 annually. It is thus seen that the average annual rate of increase is very nearly uniform, viz. about 290 or 300 children per 1000 married women. It would thus appear that the initial rate of fertility is maintained at all events for 15 years of marriage by women marrying at under 25 years of age. Whilst such a rate as that prevails in a community I do not think

FIG. 7. Distribution of Fertility amongst Women—Married at Age Under 25. Experience of New South Wales Census 1901.

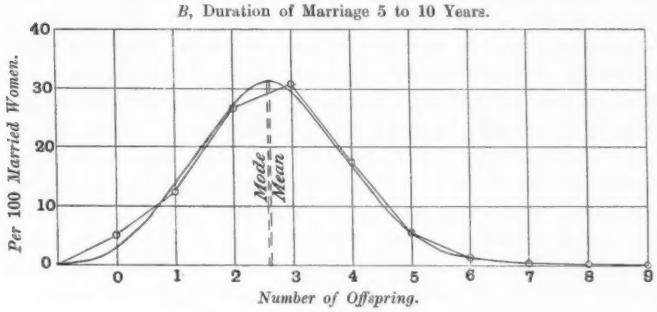
A, Duration of Marriage 4 to 5 Years.



Mean at 1·7706 Children.
Mode at 1·8631 Children (Origin of Curve).

Equation to Curve $y = 45{\cdot}9164\left(1+\frac{x}{6{\cdot}40540}\right)^{18{\cdot}85225}\left(1-\frac{x}{3{\cdot}66295}\right)^{10{\cdot}78065}$.

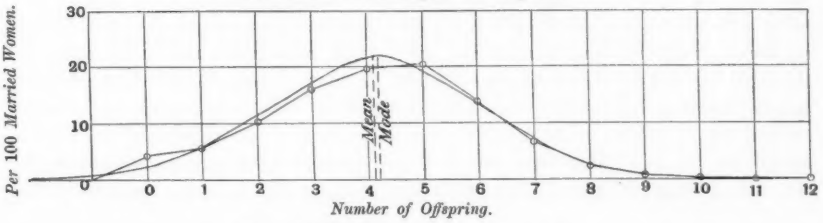
B, Duration of Marriage 5 to 10 Years.



Mean at 2·6348 Children.
Mode at 2·5932 Children (Origin of Curve).

Equation to Curve $y = 30{\cdot}9904\left(1+\frac{x}{9{\cdot}7003}\right)^{31{\cdot}6924}\left(1-\frac{x}{12{\cdot}8453}\right)^{41{\cdot}9672}$.

C, Duration of Marriage 10 to 15 years.



Mean at 4·1229 Children.
Mode at 4·2086 Children (Origin of Curve).

Equation to Curve $y = 20{\cdot}2118\left(1+\frac{x}{10{\cdot}0752}\right)^{10{\cdot}86664}\left(1-\frac{x}{8{\cdot}2141}\right)^{8{\cdot}85926}$.

it can be seriously held that there has been very much Malthusian restraint; especially is this so when it is borne in mind that the potential embryonic life has not of course been taken into account. It may therefore I think be held that the distributions of fertility as set forth in Tables XV. and XVI. are as nearly as possible those of a non-restricting population.

C. *On the Correlation between Age at Marriage and Duration of Life beyond Age* 46.

Although in Section A of this investigation we found that the number of offspring born to a woman is highly correlated with the duration of life beyond the reproductive period, irrespective of the age at marriage, yet it is possible that the fact that maximum fertility is not favourable to extreme longevity may be due to the generally early contraction of marriages, which is of course generally productive of large families—before the proper development of the reproductive organs. The period of mature or maximum development has been shown elsewhere to be about age 25*. Although it is clear that early marriage has not the effect of arresting development (perhaps it tends to too rapid development), this may be accomplished to the subsequent prejudice of life. Had it been possible to have examined each age at marriage with each sized family and subsequent life incidental to each sized family, this could readily have been ascertained, but the voluminous statistics involved present an almost insuperable obstacle to its accomplishment. However, statistics are published in the "Vital Statistics" of New South Wales for the two years 1901 and 1902 which show the age at marriage, the number of offspring, and the duration of life after marriage, but we are unable to connect mothers of each sized family with the duration of marriage. One table in the "Vital Statistics" shows the ages at marriage in quinquennial groupings, together with the durations of marriage by single years and the total offspring. In order to obtain from this table the average number of children born to women marrying in each quinquennial group, and who survived the reproductive period, I have taken the earliest duration such that if added to the latest year of such group the sum would be 49. Thus for age at marriage under 20 the duration of marriage was 29 years and upwards; for age at marriage 20—25 years the duration was 24 years and upwards; and so on. From the table referred to I have compiled the information shown in columns 2, 3 and 4 in Table XVII. Another table in the "Vital Statistics" shows the ages at marriage in single years and the duration of marriage also in single years, but not the number of offspring, and from this table I have computed for each single year of age at marriage the mean duration of life beyond age 46, and the results appear in Table XVIII. The means of the quinquennial groups are shown in the fifth column of Table XVII. Although the two sets of women shown in the

* *Biometrika*, Vol. I. Part I. p. 33.

two tables are not necessarily the same, it will be noticed that there is but little difference in the numbers within the groups. It must also be noted that all the figures refer to age at first marriage, if more than one has been contracted.

TABLE XVII.

Age at Marriage	Number of Deaths	Total Children	Average Children	Average Life beyond Age 46 by Survivors of that Age
Under *20* years	1226	9892	8·06	20·66 years
20 and under *25* years	1661	11462	6·90	20·70 „
25 „ *30* „	843	4399	5·22	23·54 „
30 „ *35* „	326	1096	3·36	23·44 „
35 „ *40* „	121	253	2·09	25·20 „
40 „ *45* „	92	60	0·65	

TABLE XVIII.

Age at Marriage	Deaths after Age 46	Total years lived after Age 46	Mean Duration after Age 46	Age at Marriage	Deaths after Age 46	Total years lived after Age 46	Mean Duration after Age 46
15 years	57	1277	22·40 years	*28* years	156	3723	23·87 years
16 „	162	3401	20·99 „	*29* „	93	2013	21·65 „
17 „	238	4795	20·15 „	*30* „	143	3487	24·39 „
18 „	399	8203	20·56 „	*31* „	44	902	20·50 „
19 „	378	7793	20·62 „	*32* „	56	1383	24·70 „
20 „	417	8616	20·66 „	*33* „	42	915	21·79 „
21 „	395	8246	20·88 „	*34* „	39	907	23·26 „
22 „	319	6855	21·49 „	*35* „	38	928	24·42 „
23 „	272	5224	19·21 „	*36* „	29	760	26·21 „
24 „	251	5298	21·11 „	*37* „	24	593	24·71 „
25 „	269	6339	23·57 „	*38* „	21	514	24·48 „
26 „	183	4425	24·18 „	*39* „	10	279	27·90 „
27 „	132	3107	23·54 „				

From the statistics contained in Table XVIII. the correlation between the age at marriage and the duration of life after age 46 was determined. The following were the constants:

Mean Age at Marriage ..... 22·6806 years

Mean Duration of Life after Age 46 ..... 21·5942 „

Standard Deviation { Age at Marriage ..... 4·8835 „
Duration of Life ..... 1·6004 „

Correlation ..... + ·70679 ± ·00523

Coefficient of Regression ..... + ·23163

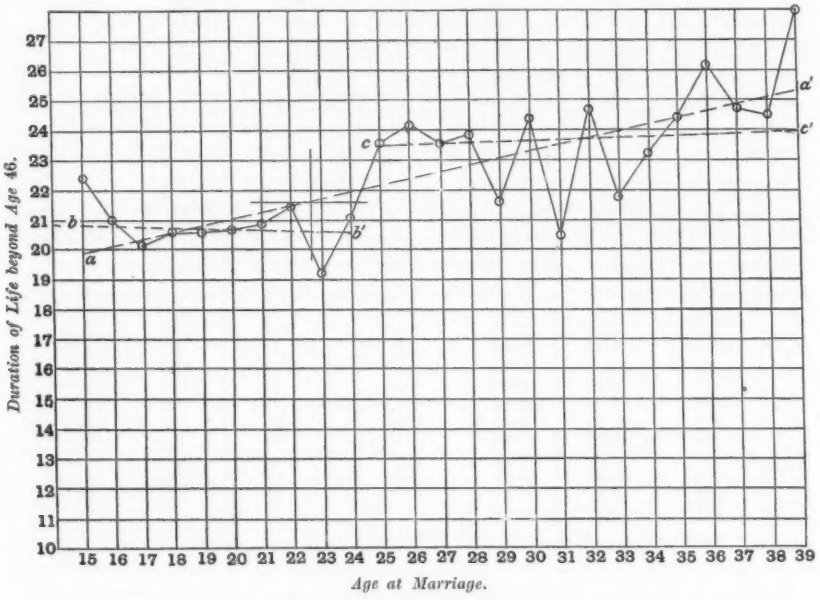
Regression Straight Line (Origin at Age at Marriage 14 years. Unit of $x = 1$ year), $y = 19{\cdot}5835 + {\cdot}23163\,x$.

The statistics and regression straight lines are shown on Fig. 8.

The correlation is high and within very small limits of probable error, and we are therefore justified in concluding that early marriage, whether in consequence of generally larger families (see Section A) or in consequence of immature development, is distinctly prejudicial to longevity beyond the reproductive period. If any weight is to be attached to the results of the comparatively meagre observations of the marriages contracted between ages 35 and 40, we would be

FIG. 8. Age at Marriage and Duration of Life beyond Age 46 by Survivors of that Age.

Females.



Equations to Regression Straight Lines:

aa' (Origin at 14 years of Age. Unit of $x=1$ year)

$$y = 19{\cdot}5835 + {\cdot}23163x.$$

bb' (Origin at 14 years of Age. Unit of $x=1$ year)

$$y = 20{\cdot}8118 - {\cdot}0228x.$$

cc' (Origin at 24 years of Age. Unit of $x=1$ year)

$$y = 23{\cdot}4057 + {\cdot}05629x.$$

further justified in concluding that the longer marriage is deferred the greater the probable lifetime after the reproductive period is passed. If, however, we examine Fig. 8, we see that the statistics divide into two fairly well defined sections, viz. from age 15 at marriage to age 24, and from age 25 upwards. The dividing age (24—25) is remarkable as that found to be that of maximum initial

fertility and also that of greatest physical development, at least so far as stature is concerned*. I have therefore determined the correlation for each of these sections with the following results:

	Age at Marriage	
	15 to 24 years	*25 to 39 years*
Mean Age at Marriage	20·0104 years	28·7099 years
Mean Duration of life beyond Age 46	20·6745 „	23·6708 „
Standard Deviation { Age at Marriage	2·3557 „	3·5277 „
Standard Deviation { Duration of Life	·6221 „	1·0769 „
Correlation	$-·0865 \pm ·0124$	$+·1844 \pm ·01822$
Coefficient of Regression ...	$-·02284$	$+·05629$
Regression Straight Line ...	Origin at Age at Marriage 14 years $y = 20·8118 - ·0228x$	Origin at Age at Marriage 24 years $y = 23·4057 + ·05629x$

In these two series we see that the correlation is low, though sensibly within the limits of probable error. The coefficient of regression shows that there is little difference in the average duration of life beyond the reproductive period by marriage at any age under 25 years—the mean being 20·01 years; also that there is but little difference by marriage at any age between 25 and 39—the mean of which is 23·67 years. But there is a most decided advantage in longevity by deferring marriage until after age 25—a difference of $3\frac{2}{3}$ years—and as these are the results of over 4000 observations, they must be accepted as reliable and not merely as accidental. In comparing these results with those of Section A, it must be remembered that in these observations the average lifetime is curtate, i.e. only completed years have been counted—the year of death is not considered—and that the age 46 is the origin. To bring these into line, therefore, with those of Section A, $1\frac{1}{2}$ years must be added to the present results. We see from Table XVII. that the average number of offspring to marriages contracted under age 25, and in which the wife at least survives to age 46, is 7·40, and that of marriages contracted between ages 25 and 40, 4·46. If now we turn to Fig. 3 of Section A we find from the parabola that the average durations of life after age 45 to mothers of 7·40 and 4·46 children are almost identical, viz. 23·4 years, whilst (adding the corrections referred to above, viz. $1\frac{1}{2}$ years) the average durations of those who marry before and after 25 years of age are 22·2 and 25·2 years respectively. These differences I think show that the age at marriage, as well as the number of offspring, is a factor in connection with the duration of life beyond the reproductive period. During the reproductive period it is of course obvious that as each successive birth has its risk, mothers of large families imperil their lives more than those of smaller ones—not simply in proportion to the number of births, but after the first confinement in a constantly increasing ratio. From the "Vital Statistics" of New South Wales, I have been enabled to determine the risk incidental to each confinement. In one table is to be found the order of confinement—first, second, etc., whilst in another are to be found the

* *Biometrika*, Vol. I. Part I. p. 33.

deaths in childbirth in order of confinement. This information is available for the nine years 1894 to 1902 and, together with the crude probability of death in each confinement, is shown in Table XIX.

TABLE XIX.

Probability of Death in Confinement. Experience of New South Wales, 1894—1902.

Order of Confinement	Births	Deaths in Confinement	Probability of Death in Confinement	Order of Confinement	Births	Deaths in Confinement	Probability of Death in Confinement
1	65778	552	·00839	*11*	4698	46	·00979
2	52818	258	·00488	*12*	2716	34	·01252
3	43462	240	·00552	*13*	1427	24	·01682
4	35510	213	·00600	*14*	703	9	·01280
5	28920	204	·00705	*15*	338	6	·01775
6	23527	161	·00684	*16*	129	1	·00775
7	18729	130	·00694	*17*	68	1	·01471
8	14659	132	·00900	*18*	21	2	·09524
9	10580	114	·01078	Beyond *18*	18	—	—
10	7230	61	·00844		236	4	·01695

Rejecting the statistics of the sixteenth and subsequent confinements as being too limited to yield satisfactory results, the probabilities of death were graduated by means of a parabola of the fourth order, the equation to which was found to be

$$y = 8{\cdot}384571 + {\cdot}846415\,x + {\cdot}040384\,x^2 - {\cdot}002525\,x^3 + {\cdot}000568\,x^4,$$

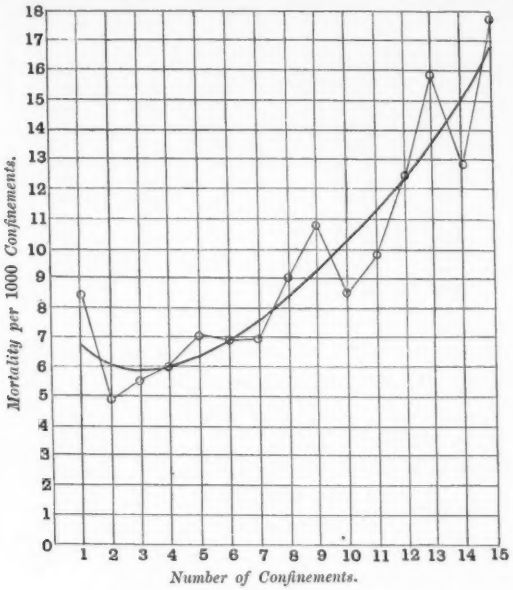
where y = the probability of death, and x the number of confinement—the origin of the curve being at the eighth confinement. There are no points of inflexion, and the curve is convex throughout. The theoretical minimum occurs at the 3·0954 confinement. The statistics and curve will be found plotted on Fig. 9. Obtaining the probabilities of death from the parabola, we are enabled to ascertain the number of survivors of any given number of fertile married women at each confinement—presuming the mortality of childbirth to be the only operative cause of death. Thus adopting the symbols used in life tables*, calling x the number of confinement, q_x the probability of death in the xth confinement, p_x the probability of surviving it, and l_x, we have (taking 10,000 fertile married women):

x	q_x	p_x	l_x	x	q_x	p_x	l_x
1	·00667	·99333	9933	*9*	·00927	·99073	9387
2	·00604	·99396	9873	*10*	·01027	·98973	9291
3	·00583	·99417	9815	*11*	·01127	·98873	9186
4	·00595	·99405	9757	*12*	·01240	·98760	9072
5	·00632	·99368	9695	*13*	·01367	·98633	8948
6	·00691	·99309	9628	*14*	·01511	·98489	8831
7	·00758	·99242	9555	*15*	·01679	·98321	8665
8	·00838	·99162	9475				

* *Institute of Actuaries Text Book*, Part II, p. xxi.

Thus if no other causes of death were in operation, we see that no less than 134 out of every 1000 women would perish from accidents of childbirth in accomplishing the fifteenth confinement, and each subsequent confinement would be accompanied by an ever-increasing risk. It thus appears that granting that economic reasons (such as in sparsely populated territories as Australia, where there is but little more than one individual to the square mile) demand that there

Fig. 9. Probability of Death in Confinement. Experience of New South Wales, 1894—1902.



Equation to Curve (Origin at 8 Children):

$$y = 8{\cdot}384571 + {\cdot}846415x + {\cdot}040384x^2 - {\cdot}002525x^3 + {\cdot}000568x^4.$$

should be but little if any restriction in families, it is distinctly prejudicial to the individual, both during and beyond the reproductive period, to bear a large family. It may of course be urged that it is prejudicial within the reproductive period even to bear but one, but it will be seen from the last table that to bear a family of six the risk cannot be said to be large—over 96 per cent. of the women will survive this—whilst the reproductive period passed almost a maximum expectation is attained.

Turning now to married men marrying at various ages, I have computed from similar tables to those from which the information was obtained for married women, the mean duration of life beyond age 46 by married men surviving that age and marrying at the various ages. The age of 46 is not of course a critical age in man as it is in woman, but has been adopted so that comparison

might be effected with the duration of life in women. The results are shown in Table XX.

TABLE XX.

Mean Duration of Life of Married Men beyond Age 46 by survivors of that age. Experience of New South Wales, 1901 and 1902.

Age at Marriage	Deaths after Age 46	Total years lived after Age 46	Mean Duration of life after Age 46	Age at Marriage	Deaths after Age 46	Total years lived after Age 46	Mean Duration of life after Age 46
20	222	4290	19·32 years	*33*	137	3075	22·45 years
21	342	6595	19·28 „	*34*	158	3152	19·95 „
22	358	7021	19·61 „	*35*	136	3407	25·05 „
23	336	6510	19·38 „	*36*	109	2409	22·10 „
24	342	6712	19·63 „	*37*	91	1937	21·29 „
25	410	7958	19·41 „	*38*	86	1903	22·13 „
26	351	6777	19·31 „	*39*	69	1390	20·14 „
27	300	5894	19·65 „	*40*	102	2504	24·55 „
28	339	6489	19·14 „	*41*	39	821	21·05 „
29	241	4990	20·71 „	*42*	52	957	18·40 „
30	285	5864	20·58 „	*43*	27	483	17·90 „
31	158	3223	20·40 „	*44*	35	824	23·54 „
32	180	4075	22·64 „				

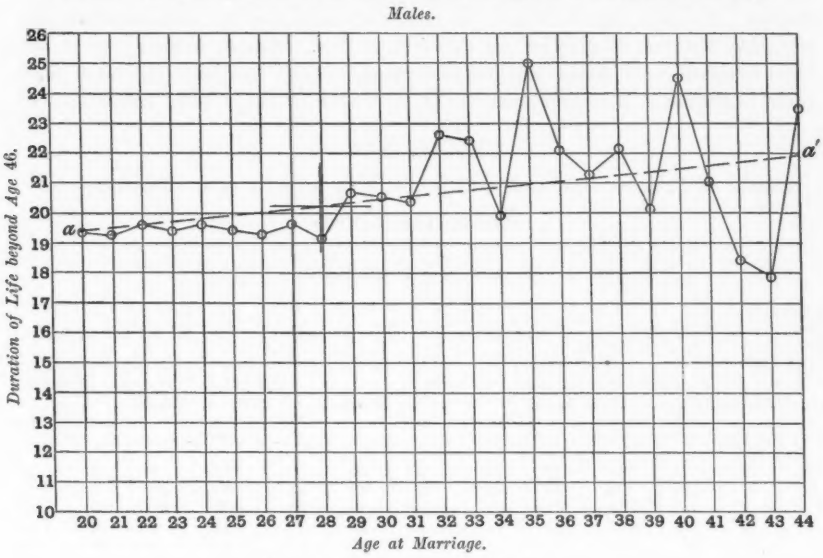
The correlation between age at marriage and duration of life beyond age 46 was determined, and also the regression straight line. The following are the general results:

Mean Age at Marriage .....27·94414 years.
Mean Duration of Life beyond Age 46 .....20·2365 „
Standard Deviation { Age at Marriage ..... 7·1415 „ / Duration of Life ..... 1·4623 „ }
Correlation ..... + ·51973 ± ·00703
Coefficient of Regression ..... + ·10642
Regression Straight Line (Origin at Age at Marriage 19 years. Unit of $x = 1$ year), $y = 19·2847 + ·10642\,x$.

The statistics and regression straight line will be found plotted on Fig. 10. Here we see that early marriage is again apparently unfavourable to longevity in man, though not so pronounced as in woman, as perhaps *a priori* might have been suspected. It has been suggested that two causes operate against longevity in excessively fertile women, viz. the physical strain in bearing and the mental strain in rearing large families. The latter factor only is operative in the case of males, and probably contributes to the shortening of the duration of life in men marrying at the early ages. Again we see that the statistics divide into two well-defined series, the demarcation being at age 28 at marriage. Up to and inclusive of that age the results are remarkably uniform, the mean of the mean durations of life beyond age 46 being 19·41 years. For marriages contracted beyond age 28 the results are rather irregular, but distinctly exhibit an increase

in longevity, the mean duration being 21·53 years—an advance of over 2 years—and there can be no doubt that the increase is a real one, whatever may be the cause. It of course may be urged, and with much force, that amongst those following hazardous occupations, such as the industrial classes, marriage is generally

Fig. 10. Age at Marriage and Duration of Life beyond Age 46 by Survivors of that Age.



Equation to Regression Straight Line:

aa' (Origin at 19 years of Age. Unit of $x=1$ year)

$$y = 19{\cdot}2847 + {\cdot}10642x.$$

contracted early in life, whilst amongst those following the least hazardous—the professional—marriage is contracted much later. This is true, as will be seen from Section E, though the difference in age at marriage does not exceed $1\frac{2}{3}$ years in the two classes. The adults' death rates are also shown in the same section, that of the professional class being 15·73 and that of the industrial 22·72 per 1000 exposed to risk. But notwithstanding this great difference in the mortality in view of the slight difference in the age at marriage, it is doubtful whether if all allowances be made the difference in the duration of life can be thus accounted for. Some portion is probably due to the struggle to maintain the large family generally connected with early marriage*.

[* It will be seen from *Biometrika*, Vol. I. p. 46 *et seq.*, that a man's prime is about 28 in the same sense as a woman's is about 24. May not therefore the lessened expectation of life in the case of man as in the case of woman be due to exercise of sex-functions before the prime? Further is not the association of extreme fertility with shorter duration of life in women due in part to the fact that very large families mean as a rule early marriages? K. P.]

D. *On the Correlation between Duration of Marriage of Females and Number of Offspring—at various ages at Marriage.*

There are two methods by which the necessary information may be obtained: first, by noting at death the date of and age at marriage and the total offspring; second, at a census obtaining the information direct from the living married women. In New South Wales the materials requisite for both methods are available, but the information for the first method refers to women who at the time of death were either married or widows, or who had been separated from their husbands. This information will be found in the "Vital Statistics" of

TABLE XXI.

Deaths of Females, Age at Marriage, Duration of Life after Marriage, and Number of Offspring.

Duration of life after Marriage (years)	Age at Marriage											
	Under 20			20 and under 25			25 and under 30			30 and under 35		
	Deaths	Offspring		Deaths	Offspring		Deaths	Offspring		Deaths	Offspring	
		Total	Average		Total	Average		Total	Average		Total	Average
0	11	7	·64	26	13	·50	13	8	·62	5	4	·80
1	17	13	·76	66	53	·80	41	34	·83	10	2	·20
2	9	14	1·56	37	43	1·16	19	20	1·05	10	13	1·30
3	16	26	1·62	29	44	1·52	23	37	1·61	8	13	1·62
4	16	31	1·94	48	90	1·87	18	24	1·33	8	14	1·75
5	11	35	3·18	37	72	1·95	15	29	1·93	6	8	1·33
6	22	50	2·27	51	125	2·45	26	66	2·54	4	9	2·25
7	15	38	2·53	27	77	2·85	17	37	2·18	7	13	1·86
8	18	49	2·72	32	92	2·88	22	55	2·50	6	19	3·17
9	27	87	3·22	34	98	2·88	14	49	3·50	9	24	2·67
10	27	99	3·67	42	153	3·64	15	41	2·73	9	31	3·44
11	27	103	3·82	37	141	3·81	27	79	2·93	6	14	2·33
12	17	65	3·82	46	192	4·17	23	104	4·52	3	17	5·67
13	31	120	3·87	29	130	4·48	20	85	4·25	6	13	2·17
14	23	99	4·30	41	179	4·36	13	48	3·69	4	8	2·00
15	28	122	4·36	51	230	4·51	17	82	4·82	—	—	—
16	23	111	4·83	42	234	5·57	20	104	5·20	—	—	—
17	29	136	4·69	39	239	6·13	9	38	4·22	—	—	—
18	34	184	5·41	44	244	5·55	16	72	4·50	—	—	—
19	26	124	4·77	55	281	5·11	19	87	4·58	—	—	—
20	37	206	5·57	40	217	5·43	—	—	—	—	—	—
21	34	208	6·12	36	233	6·47	—	—	—	—	—	—
22	23	186	8·09	44	234	5·32	—	—	—	—	—	—
23	28	185	6·61	41	281	6·85	—	—	—	—	—	—
24	28	204	7·29	41	230	5·61	—	—	—	—	—	—
25	32	243	7·59	—	—	—	—	—	—	—	—	—
26	34	244	7·18	—	—	—	—	—	—	—	—	—
27	23	171	7·43	—	—	—	—	—	—	—	—	—
28	18	143	7·94	—	—	—	—	—	—	—	—	—
29	26	187	7·19	—	—	—	—	—	—	—	—	—

New South Wales for the two years 1901 and 1902, whilst that relating to married women living with their husbands at the time of the census of 1901 will be found in Part "Conjugal Condition and Families" of the Census returns. The former tables show for married women dying in those years the age at marriage (in quinquennial groups), the duration of marriage in single years, and total offspring. Limiting the information to the reproductive period, the statistics are shown in Table XXI.

The correlation and regression straight lines were determined for each of these series with the results set forth in Tables XXII. and XXIII.

TABLE XXII.

General Results.

Age at Marriage	Mean Duration of Life after Marriage (within Reproductive Period)	Mean Size of Family	Standard Deviation		Correlation. Duration of Life after Marriage and Size of Family	Regression
			Duration of Life after Marriage	Size of Family		
Under *20*	16·7648 years	4·9155	7·9165 years	2·0168	·97328 ± ·00133	·24796
20 and under *25*	12·6961 „	3·8670	7·2585 „	1·8226	·96554 ± ·00143	·24245
25 „ *30*	9·3217 „	2·8398	5·7459 „	1·4459	·94568 ± ·00362	·23797
30 „ *35*	6·8762 „	2·0000	4·1622 „	1·0963	·65700 ± ·03812	·17304

TABLE XXIII.

Regression Formulæ.

x = Duration of Life after Marriage (within Reproductive Period). y = Number of Offspring.

Age at Marriage,	Under *20* years,	$y = ·7585 + ·24796x$
„ „	*20* and under *25* years,	$y = ·7888 + ·24245x$
„ „	*25* „ *30* „	$y = ·6215 + ·23797x$
„ „	*30* „ *35* „	$y = ·8101 + ·17304x$

Note:—Origin at marriage and unit of $x = 1$ year.

The statistics and regression straight lines will be found plotted on Figs. 11, 12, 13 and 14; the average number of offspring for each year of life after marriage is shown by small circles, and that for quinquennial periods by triangles. It is seen that in each series although the statistics are somewhat irregular they are well represented by the straight lines, and thus it would appear that the initial fertility incidental to the age at marriage is preserved almost unchanged throughout the reproductive period. It must be clearly understood that the foregoing are the results of women dying who had been married and whose husbands may or may not have survived them. Although it is seen that the initial fertility is preserved throughout the whole of the reproductive period, yet the age at marriage is the determinant of that fertility. Up to age 30 there would seem to be but little difference in this initial fertility, although it is slightly less at

Age at Marriage, Duration of Marriage and Number of Offspring. Experience of New South Wales, Deaths of Married Women, 1901 *and* 1902.

FIG. 11. Age at Marriage, Under 20 years.

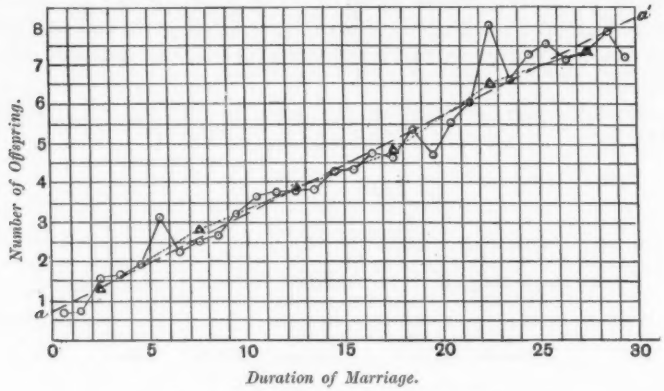


FIG. 12. Age at Marriage, 20 to 25 years.

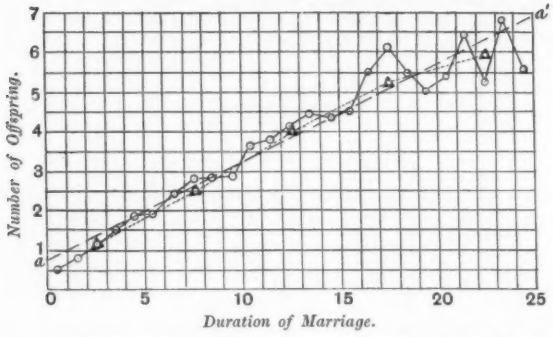


FIG. 13. Age at Marriage, 25 to 30 years.

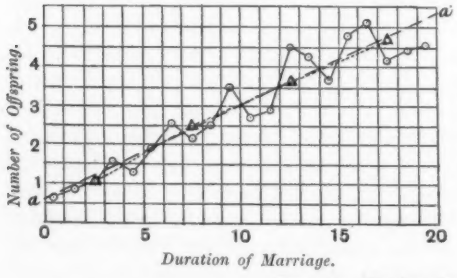
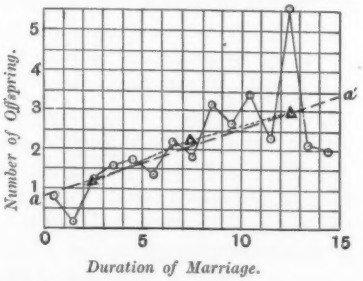


FIG. 14. Age at Marriage, 30 to 35 years.



Regression Straight Lines *aa'*.
Averages of Quinquennial Period shown thus Δ.....Δ.....

each advance in the age at marriage. At age 30—35 there is a most decided fall in the initial fertility, and the extent of the fall when marriage is deferred to age 35—40 may be judged from the fact that 20 women who married at that age, and whose duration of marriage was under 5 years, bore only 14 children, and 7 whose duration was 5 to 10 years bore 10 children. After age 40 at marriage—by unprejudiced* spinsters—the initial fertility may be said to have reached zero. Calculating the probable average number of children at the end of the reproductive period (end of forty-fifth year) by means of the regression formulæ, we obtain

For Age at Marriage under 20 .....7·55
" " " " 20 and under 25.....6·51
" " " " 25 " " 30.....5·12
" " " " 30 " " 35.....3·21

—results which are somewhat less than the actual results of those women who survived the reproductive period, and which are shown in Table XVII.

Turning now to the census results of New South Wales for 1901, we are able to obtain—for wives who were living with their husbands on the night of the census—the age, the duration of marriage, and the number of offspring. From the nature of the tabulation we are unable to obtain this information for all wives, but this will best be understood from the following specimen of the tabulation:

SPECIMEN OF TABULATION.

Number of Wives without Issue.

Age of Wife	Total Wives	Duration of Marriage								
		Under 1 year	1 year	2 years	3 years	4 years	5 and under 10 years	10 and under 15 years	15 and under 20 years	And so on in quinquennial periods
14 years										
15 "										
16 "										
&c. "										
20 "										
21 and under *25*										
25 " *30*										
30 " *35*										
&c. &c.										

These tabulations appear for each sized family up to 10 children, 11 and over being amalgamated. Thus if we wish to trace women married, say, at age under 20, and with duration of marriage under 5 years, we can only make use of those who were specifically stated at the census to be under 20 and who had been married less than 5 years. This of course does not give us all such women, since

* Those for whom conception was not antenuptial.

some who had been married under 1 year, and between 1 and 2 years, and were aged 20 at census, belong to the age at marriage group under 20. So also do some women aged 21 to 25 years, and who had been married 2 and under 5 years. If we take duration of marriage 5 to 10 years with age at census 20—25, we see that the whole of these women must have been married at under age 20. But so too do some who were aged 25—30 with the same duration of marriage, but we are unable to determine how many. I have therefore been obliged to confine the investigation to such groups as are absolutely certain. Another difficulty has to be faced when we reach the duration of marriage sufficient to yield 11 or more children. To effect the distribution of families beyond 10

TABLE XXIV.

Distribution of Fertility amongst Wives Married at Age under 20 Years. Experience of New South Wales Census, 1901.

Number of Children in Family	Duration of Marriage											
	Under 5 years		5 and under 10 years		10 and under 15 years		15 and under 20 years		20 and under 25 years		25 and under 30 years	
	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children
0	1171	—	166	—	120	—	89	—	68	—	44	—
1	1166	1166	438	438	228	228	158	158	77	77	41	41
2	188	376	1026	2052	376	752	195	390	88	176	63	126
3	22	66	1034	3102	624	1872	289	867	152	456	65	195
4	—	—	422	1688	812	3248	429	1716	170	680	83	332
5	—	—	83	415	864	4320	490	2450	231	1155	105	525
6	—	—	17	102	484	2904	583	3498	312	1872	141	846
7	—	—	4	28	155	1085	520	3640	330	2310	167	1169
8	—	—	—	—	39	312	398	3184	396	3168	237	1896
9	—	—	—	—	9	81	170	1530	367	3303	272	2448
10	—	—	—	—	1	10	72	720	314	3140	333	3330
11	—	—	—	—	—	—	36	396	260	2860	271	2981
12	—	—	—	—	—	—	—	—	34	408	208	2496
13	—	—	—	—	—	—	—	—	—	—	131	1703
14	—	—	—	—	—	—	—	—	—	—	77	1078
15	—	—	—	—	—	—	—	—	—	—	46	690
16	—	—	—	—	—	—	—	—	—	—	19	304
17	—	—	—	—	—	—	—	—	—	—	9	153
18	—	—	—	—	—	—	—	—	—	—	6	108
19	—	—	—	—	—	—	—	—	—	—	3	57
20	—	—	—	—	—	—	—	—	—	—	1	20
Totals	2547	1608	3190	7825	3712	14812	3429	18549	2799	19605	2322	20498
Average Family	·63		2·45		3·99		5·41		7·00		8·83	
Per cent. without issue	46·0		5·20		3·23		2·60		2·43		1·90	

children, we are obliged to call to our assistance another table in the census which shows the number of wives with each sized family and the ages at marriage, but not the duration of marriage. In the manner thus shown—having regard to the age at marriage—the distribution of families of 11 and more children has been effected, and to this extent the columns of the last three durations in Table XXIV. are vitiated, though the averages obtained cannot possibly be affected to any appreciable extent. This table shows the distribution of fertility to wives married at under 20 years for various durations of marriage within the reproductive period.

The average issue for under 5 years' duration of marriage is evidently too small and the percentage of women without issue too large, owing to the large number of women married under 1 year to be fair criteria of the results of a mean duration of $2\frac{1}{2}$ years' marriage. The results of this duration were obtained from the census and were found to be 422 women with 468 children (56 had 0, 270 had 1, 90 had 2, and 6 had 3), or an average of 1·11 each with 13·3 per cent. without issue. As, however, the result was 1·32 in the case of deaths of married women, which I think is more probably the truth in view of the fact that so many of those marrying under 20 being undetermined in the ages 20 and 21—25, I adopted this value in the calculation of the correlation and regression straight line, with the following general results:

Mean Age at Marriage		14·6415 years.
„ Size of Family		4·6967 children.
Standard Deviation	Duration of Marriage	7·9585 years.
	Size of Family	2·3841 children.
Correlation		·99790 ± ·00002
Coefficient of Regression		·29894

Regression Straight Line (Origin at Marriage. Unit of $x = 1$ year)

$$y = ·3198 + ·29894\,x,$$

where x = duration of marriage, and y the number of offspring.

The statistics and regression straight line are plotted on Fig. 15, on which is also plotted for the purpose of comparison the regression straight line as determined by deaths.

The methods of tabulation of the census do not admit of tracing the marriages contracted between intervals of 5 years, say, between 20 and 25, but does admit of them being dealt with as under 25 years at marriage*. The information for

* It might at first sight be thought possible to have effected the desired computations for quinquennial periods. For the results are ascertainable for marriages contracted as under each quinquennial age: as for instance, under age 20, under age 25, under age 30, &c., and the difference between any two succeeding ages would appear to give the necessary results. Thus, take the duration of marriage 5 and under 10 years, marrying under age 20 there were 3190 such women who had 7825 children; marrying under age 25 there were 18,086 such women who had 47,653 children; but it is not true that the difference between these two series, viz. 14,896 women having 39,828 children married between the ages 20 and 25, for many (and it is impossible to say how many) of the latter were married at ages under 20. The number shown as marrying under 20 are those we are certain of from the returns but are not all. The same applies to all the groups.

the latter period I have also taken from the census returns, and the results are shown in Table XXV.

TABLE XXV.

Distribution of Fertility amongst Wives Married at Age under 25 Years. Experience of New South Wales Census, 1901.

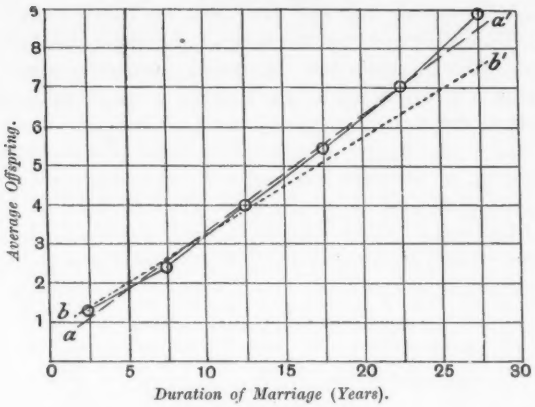
Number of Children in Family	Duration of Marriage									
	Under 5 years		5 and under 10 years		10 and under 15 years		15 and under 20 years		20 and under 25 years	
	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children	Wives	Total Children
0	6152	—	1094	—	757	—	553	—	332	—
1	8133	8133	2230	2230	1039	1039	637	637	344	344
2	3668	7336	4811	9622	1874	3748	928	1856	477	954
3	597	1791	5567	16701	2931	8793	1402	4206	669	2007
4	43	172	3124	12496	3603	14412	1967	7868	858	3432
5	2	10	996	4980	3715	18575	2235	11175	1122	5610
6	—	—	226	1356	2542	15252	2662	15972	1383	8298
7	—	—	36	252	1233	8631	2570	17990	1548	10836
8	—	—	2	16	422	3376	2063	16504	1735	13880
9	—	—	—	—	118	1062	1125	10125	1670	15030
10	—	—	—	—	38	380	539	5390	1368	13680
11	—	—	—	—	7	77	200*	2200	652†	7172
12	—	—	—	—	—	—	90*	1080	454†	5448
13	—	—	—	—	—	—	50*	650	264†	3432
14	—	—	—	—	—	—	29*	406	149†	2086
15	—	—	—	—	—	—	—	—	74†	1110
16	—	—	—	—	—	—	—	—	36†	576
17	—	—	—	—	—	—	—	—	15†	255
18	—	—	—	—	—	—	—	—	8†	144
Totals	18595	17442	18086	47653	18279	75345	17050	96059	13158	94294
Average Family	·94		2·64		4·12		5·64		7·17	
Per cent. without issue	33·1		6·05		4·14		3·24		2·52	

Owing to the number of wives (4114) who had been married under 1 year, the statistics of under 5 years' duration of marriage were not used to determine the average number of offspring for the mean of that duration, but the actual result of 2½ years' duration, viz. 3968 wives (563 without issue, 2570 with 1 child, 814 with 2, and 21 with 3) having 4261 children, or an average of 1·07. The percentage of childless women was 14·2. With this substitution the correlation

* Estimated—Total 11 and over, 369.

† Estimated—Total 11 and over, 1652.

FIG. 15. Women Married under 20.



Regression Straight Lines:

aa' (Census), $y = \cdot3198 + \cdot29894x$ } Origin at Marriage.
bb' (Deaths), $y = \cdot7585 + \cdot24796x$ } Unit of $x = 1$ year.

between duration of marriage and number of offspring was determined from the figures in the above table with the following results:

Mean Duration of Marriage		13·7295 years.
„ Size of Family		4·5031 children.
Standard Deviation	Duration of Marriage	5·9345 years.
	Size of Family	1·7972 children.
Correlation		1·0000
Coefficient of Regression		·30286

Regression Straight Line (Origin at Marriage. Unit of $x = 1$ year)

$$y = \cdot3450 + \cdot30286\,x,$$

where x = duration of marriage, and y the number of offspring.

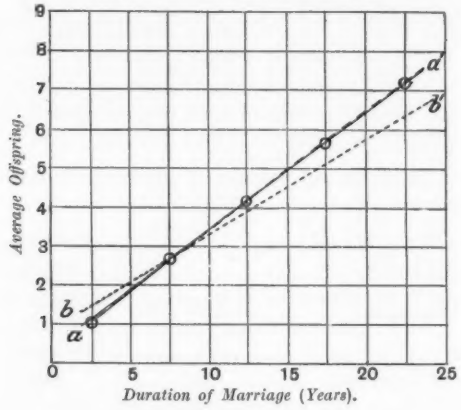
For the purpose of comparison the regression straight line was determined from the deaths on a similar basis (age at marriage under 25 years, and duration of marriage in quinquennial groupings), and was found to be $y = \cdot7360 + \cdot2484\,x$.

On comparing the results from the census figures of those marrying at under 20 with those marrying under 25, we see that in both cases, as in the results obtained from the deaths, that the initial fertility is preserved throughout the reproductive period, and that it is slightly greater when marriage is contracted under age 25 than when contracted under age 20; that is, that it is greater at age 20 to 25 than under 20. This is in accordance with results arrived at elsewhere* in connection with first births. It is also seen that the mean annual

* *Biometrika*, Vol. I. Part I. p. 33.

births are about 300 per 1000 married women marrying under 25 years. But if marriage be delayed beyond that age, not only is the initial fertility reduced, but it apparently diminishes with the duration of marriage. As has already been shown, it is impossible to obtain from the census returns the results of marriages within quinquennial groups of age at marriage, but it can be obtained in decennial groupings, though, for reasons previously set forth, not for all such marriages. Thus, those women whose ages at the census were between 30 and 35 years, and whose duration of marriage was under 5 years, must have been married between the ages of 25 and 35, as also those who were aged 35 to 40, and had been married 5 to 10 years, and so on. In this way was obtained the information relating to women marrying at ages 25 to 35, and also upon the same principle that of women marrying at ages 35 to 45. The results are shown in Table XXVI.

FIG. 16. Women Married under 25.



Regression Straight Lines:

aa' (Census), $y = \cdot3450 + \cdot30286x$ } Origin at Marriage.
bb' (Deaths), $y = \cdot7360 + \cdot24840x$ } Unit of $x = 1$ year.

It would now be well to show side by side the average families and proportion of wives without issue for each age at marriage and each duration of marriage. This is done in Table XXVII. The second column in that table shows the mean age at marriage of those marrying within the groups shown in the first column according to the experience of the years 1897 to 1902, when the ages were shown in single years.

These results are shown graphically in Figure 17—except for marriages under 20. The table shows in a striking manner the effects of delayed marriage and how the generative organs in females tend to become atrophied through non-use. For marriages contracted under age 25 we see that only $2\frac{1}{2}$ per cent. are sterile at the end of the reproductive period, whilst for marriages contracted between 25 and 35 or at a mean age of 28·4 years the percentage is nearly 12, and for ages 35 to 45

TABLE XXVI.

Distribution of Fertility amongst Wives Married at Ages 25 to 35, and 35 to 45. Experience of New South Wales Census, 1901.

Size of Family	Age at Marriage 25 to 35 years					Age at Marriage 35 to 45 years		
	Duration of Marriage					Duration of Marriage		
	2½ years	Under 5 years	5 to 10 years	10 to 15 years	15 to 20 years	2½ years	Under 5 years	5 to 10 years
0	312	1879	654	445	281	133	619	345
1	614	1938	548	293	162	41	193	100
2	139	1220	879	400	232	2	59	81
3	1	217	953	510	277	1	16	30
4	2	30	644	517	311	—	—	14
5	—	—	261	448	328	—	—	5
6	—	—	80	323	302	—	—	2
7	—	—	20	198	216	—	—	—
8	—	—	2	65	139	—	—	—
9	—	—	—	20	72	—	—	—
10	—	—	—	3	25	—	—	—
11	—	—	—	2	11	—	—	—
12	—	—	—	1	6	—	—	—
13	—	—	—	1	3	—	—	—
14	—	—	—	—	1	—	—	—
15	—	—	—	—	1	—	—	—
Total Wives	1068	5284	4041	3226	2367	177	887	577
Total Children	903	5149	9682	11032	9936	48	359	445
Average Family	·85	·97	2·39	3·42	4·20	·27	·40	·77
Per cent. without issue	29·21	35·56	16·18	13·79	11·87	75·14	69·79	59·79

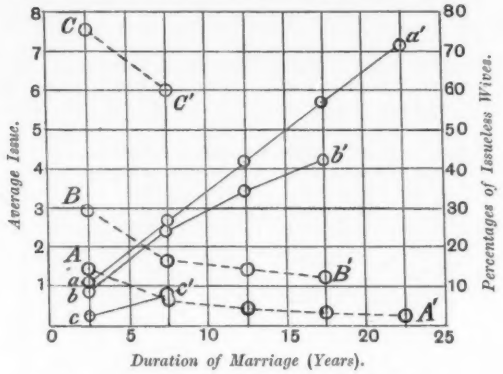
TABLE XXVII.

Average Families and Issueless Wives.

Age at Marriage (Years)	Mean Age at Marriage (Years)	Duration of Marriage													
		2½ years		Under 5 years		5 to 10 years		10 to 15 years		15 to 20 years		20 to 25 years		25 to 30 years	
		Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %	Average Family	Without Issue %
Under *20*	18·6	1·10	12·7	1·03	46·0	2·45	5·20	3·99	3·23	5·41	2·60	7·00	2·43	8·83	1·90
" *25*	21·4	1·07	14·2	·94	33·1	2·64	6·05	4·12	4·14	5·64	3·24	7·17	2·52	—	—
25 to *35*	28·4	·85	29·2	·97	35·6	2·39	16·18	3·42	13·79	4·20	11·87	—	—	—	—
35 to *45*	38·4	·27	75·1	·40	69·8	·77	59·79	—	—	—	—	—	—	—	—

or a mean of 38·4 years nearly 60 per cent. are sterile. Although it is noticeable that for marriages contracted under 25 years the average offspring is greater for each duration of marriage than those contracted under 20—except for the duration 2½ years—yet the percentage of issueless wives is also greater. This I think more an apparent than a real physiological fact and is due to the larger proportion of

FIG. 17. Average Issue and Proportion of Issueless Wives. Average Issue shown by Firm Lines. Issueless Wives shown by Broken Lines.



Age at Marriage, Under 25, Lines *AA'* and *aa'*.
" " 25—35 " *BB'* and *bb'*.
" " 35—45 " *CC'* and *cc'*.

prejudiced marriages in the younger group—and to this is also due the slightly larger average offspring in the 2½ years' duration of marriage. Taking the six years 1897 to 1902 there were 9490 marriages and 3910 antenuptial conceptions, or 41·2 per cent. of prejudiced marriages in the age group under 20, whilst of the 26,453 marriages contracted between 20 and 25 there were 7899 antenuptial conceptions or 29·9 per cent. Nor does this represent the full difference, for some women who were aged 19 at marriage, were 20 at the birth of the child and consequently were included in the later group. If adjustment be made the result will of course intensify the difference. This excess of antenuptial conception in the lower age group must amply account for the slightly higher proportion of issueless wives in the group under 25, but the fact that the average offspring is greater in the latter group than in the former—notwithstanding the greater proportion of sterile women—proves conclusively that the maximum fertility—initial and continuous—lies somewhere between age at marriage 20 and 25. Of course the test of fertility is not in the number of offspring produced during the reproductive period—for that will be greatest at the earliest age at marriage—but the rapidity with which they are produced.

A favourite method amongst statisticians and others of comparing birthrates (at least amongst those who have been able to comprehend the fallacies in the

popular method of estimating the births per 1000 of the total population) has been to show the proportion of legitimate births per 1000 married women under 45 years of age. But it is evident from Table XXVII. that this method is liable to yield misleading comparisons, for the kinetic fertility is dependent not only upon the age of the woman but also upon the age at marriage. This objection of course also applies even when the more accurate measure of determining the movements in the birthrate is adopted, viz. when the rate is determined not in one group under age 45, but in quinquennial groups within that group. In New South Wales there has been a decline in each quinquennial group except at age under 20 since 1881, as will be seen from the following statement*:

Ages of Wives	1871 Birthrate per cent.	1881 Birthrate per cent.	1891 Birthrate per cent.	1901 Birthrate per cent.
15—19 years	50·10	51·60	47·91	56·28
20—24 „	44·15	45·79	41·63	39·70
25—29 „	40·75	40·52	35·37	29·87
30—34 „	33·67	33·86	29·23	22·68
35—39 „	27·04	27·36	23·63	17·25
40—44 „	13·41	12·89	11·84	8·81

The decline since 1881 is a large one and although there has been an undoubted rise in the age at marriage since that period, the decline cannot all be attributed to that rise—though probably much is so due. Prior to 1897 the ages at marriages were not tabulated—even if they had been it would not have helped us much since the marriages to which the births in 1871 and 1881 were due were for the most part contracted in Great Britain—the great majority of the adult population then being of British birth. But as has been said the decline is not wholly due to the advance in the age at marriage—it is chiefly due I think to the Law of Regression so ably formulated and demonstrated by Mr Francis Galton in "Natural Inheritance." Whilst the systems of free and assisted immigration were adopted in New South Wales the immigrants were of special physique and chiefly drawn from the two most fertile classes of the British community, viz. the agricultural and artisan. Medical examination was necessary before departure from Great Britain and thus a differentiated community was obtained. This immigration ceased about 1885, and this was followed by the smaller decline in 1891 and the larger in 1901—the standard of the fertility of the offspring of the immigrants regressing towards that of the population from which they originally sprang. What difference there will be in the average issue of women of past generations and the ultimate families of the present generation—the age at marriage being the same—it is of course impossible to say, but I think a very close estimate may be made. Take for instance age at marriage under 20. We see from Table XVII. the average family of women of past generations marrying at this age was 8·06. This is for women whose husbands may have predeceased them before the end of

* Vide *Report of Decline in the Birth-rate Commission*, p. 7.

the reproductive period or who may have been separated from them otherwise. For women of the present generation marrying at the same age and under the same conditions we may make use of the regression formula from Table XXIII.—limiting the reproductive period to age 46 and adopting the average age at marriage as 18·6 years (see Table XXVII.) we obtain 7·55. This is a reduction of about half a child per family. Adopting the same principle the averages for the present generation are for age at marriage *20—25*, 6·51, *25—30*, 5·12, *30—35*, 3·21, as against 6·90, 5·22 and 3·36 respectively for past generations. These differences are probably the extent of the regression.

I have calculated from the information contained in the English Registrar-General's Reports the average age of spinsters marrying at ages under 45 for the years 1861–62 and 1901–2. The mean age at the former period was 24·3 years; at the latter 25·1—an advance of nearly 1 year. This however hardly conveys an adequate idea of the change that has taken place in England and Wales during the last forty years in the age constitution at marriage. At the former period only 30·9 per cent. were married after what may be regarded as the critical age (so far as fertility is concerned) of 25, whilst at the latter period the percentage had increased to 39·5. Having in view the rapid fall in the initial and subsequent fertility of marriages contracted beyond age 25 it is unquestionable that some not inconsiderable portion of the decline in the birthrate in England and Wales is due to the increasing age of spinsters at marriage. In Australian age at marriage statistics distinction is not made between first and subsequent marriages.

In Victoria—which may be regarded as typical of Australia—the average age of all women marrying at ages under 45 was 24·1 years in 1875–6; in 1901–2 this had advanced to 25·6 years. At the former period 33·8 per cent. were over 25 years of age, whilst at the latter the percentage had increased to 47·3. It would thus appear that there is a general tendency to postpone marriage both in England and Australia (probably more pronounced in the latter) and thus not only diminishing the average family but as we have seen lengthening the intervals between the offspring.

So long as the average age of the female at marriage is in the vicinity of 25 and there are no further restrictions of the family than at present the community will not be a loss for natural increase. The average family will—at the present Australian rate—be between 5 and 6 children, and although this is the gross family it must be borne in mind that the infantile—as well as the subsequent—mortality is very light when compared with Great Britain and European countries. Indeed the deaths of infants form such a small percentage of the births—about 10 per cent.—that the birthrate is considerably diminished thereby. And here (although not directly connected with this section of the investigation) it may not be out of place to refer to the correlation between the birthrate and the infantile mortality—the latter being the deaths under 1 year of age per 100 births. It will readily be seen from the next table of birthrates and infantile mortalities in various countries that

there is such correlation, for it can scarcely be regarded as fortuitous that high birthrates are associated with high mortalities and low birthrates with low mortalities. Although a higher rate of mortality might probably have been anticipated where the births are numerous, yet I think the high mortality is more the cause than the effect of the high birthrate. Now according to the English and Australian statistics of those infants who die under 12 months of age 50 per cent. die within 3 months of birth, and 70 per cent. within 6 months. If infants be reared in the natural way (which is a fairly general practice in Australia) an almost complete check is given to further conception until the completion of the period of lactation. Therefore where the mortality is heavy, even though the children have been naturally fed (falling as we have seen heaviest in the first few months of life) the impediment to further conception is removed. Where artificial feeding is resorted to of course there is no obstacle to immediate conception. Therefore in countries or communities in which artificial feeding may be the rule but where through ignorance or carelessness, or a like indifference to the production of life as to its subsequent preservation prevail, a high mortality and a high birthrate should be expected; as also in those countries where artificial feeding is the general practice. Thus we find that in Sweden, where it is stated* that almost every child is nursed by its own mother, both the infantile mortality and the birthrate are the lowest in Europe (except the birthrates in France and Ireland—both of which are abnormal). The same authority asserts that in Lower Bavaria where natural nursing has become the exception the infantile mortality reaches the extraordinary high proportion of 50 per cent.

The following table shows the birthrates and infantile mortalities in various countries so far as I have been able to obtain them. France and Ireland have been intentionally omitted—the former because the low birthrate (associated with high mortality) is well known to be due to restrictive measures, and the latter because the low birthrate (associated in this case with low mortality) is due to the constant emigration of adults of reproductive ages. Russia has also been omitted in consequence of the want of reliable figures, but it is generally understood that both the birthrate and infantile mortality are the highest in the civilized world:

Country	Birthrate	Infantile Mortality	Country	Birthrate	Infantile Mortality
Saxony ...	39·9	28·3	Japan	29·8	15·2
Hungary ...	38·9	25·6	Scotland	29·8	12·2
Bavaria ...	37·0	27·9	England and Wales	29·0	15·7
Austria ...	37·0	24·7	Belgium	28·9	16·3
Prussia ...	36·4	20·8	Switzerland ...	28·6	15·9
Italy ...	33·5	19·0	Australia	27·1	11·0
Holland ...	32·0	17·5	Sweden	26·8	10·7
Norway ...	30·2	9·6	New Zealand ...	25·7	8·1
Denmark .	29·9	13·6			

* Hugh R. Jones, M.A., M.D., B.Sc., *Journal of the Royal Statistical Society*, March 1894.

From these figures the correlation between infantile mortality and birthrate was determined. The general results were:

Mean Birthrate		31·79
„ Mortality		17·18
Standard Deviation	Birthrate	4·3328
	Mortality	6·1795
Correlation		·93369 ± ·00210
Coefficient of Regression		·6547

Regression Straight Line (x = Infantile Mortality, y = Birthrate, Origin of x, Inf. Mort. = 0) $y = ·6547x + 20·5423$, the probable error of y being ± 1·046.

The next table shows the number of survivors of age 5 per 1000 births in various countries *:

Country	Survivors	Country	Survivors	Country	Survivors	Country	Survivors
New Zealand ...	889	Ireland	837	Belgium ...	756	Italy ...	632
New South Wales	850	Sweden	783	Denmark...	755	Austria	614
Victoria ...	844	Scotland	780	France ...	751	Hungary	598
Norway... ...	838	England and Wales	762	Switzerland	748	Spain...	571

So great is the mortality generally in high birthrate countries that the numerical advantage to the community almost entirely disappears by the end of the fifth year, reckoning per 1000 of the total population. Reducing the birthrates in the proportions given in the last table, the net birthrate at the end of the fifth year is shown in the third columns of the following table:

Country	Birth-rate	Surviving 5th year	Country	Birth-rate	Surviving 5th year	Country	Birth-rate	Surviving 5th year
Hungary	38·9	23·3	Denmark	29·9	22·6	Sweden ...	26·8	20·9
Austria	37·0	22·7	England and Wales	29·2	22·1	New Zealand	25·7	22·8
Spain ...	34·8	19·9	Belgium	28·9	21·9	Victoria ...	25·7	21·7
Italy ...	33 5	22·2	Switzerland ...	28·6	21·4	France ...	22·0	16·5
Norway	30·2	25·3	New South Wales	27·4	23·3			

It may be urged that whilst the birthrate has fallen in England and Wales during the last 15 or 20 years the infantile mortality has practically remained unchanged. Even if artificial limitation of families has become more generally the practice during that period an investigation of the birthrates and infantile mortalities of the various registration districts shows that the association between the two rates does exist and that those districts which include large cities and towns (where I should imagine the knowledge of restrictive measures would be

* I am responsible for the New Zealand, New South Wales and Victorian figures; the others have been obtained from Mulhall.

best known and the practice reach its zenith) still maintain a high birthrate associated with a high infantile mortality, whilst the essentially rural districts have generally a low birthrate and a low mortality.

These rates I have computed from the data in the Registrar-General's Report for 1902, and are shown in the following table:

Registration District	Birth-rate	Infantile Mortality	Registration District	Birth-rate	Infantile Mortality	Registration District	Birth-rate	Infantile Mortality
Durham	35·9	14·1	London	28·6	14·0	Cambridge	24·2	11·0
Monmouth	35·5	13·5	Cumberland	28·0	11·0	Huntingdon	24·1	9·4
Stafford	33·7	14·3	Worcester	27·8	11·7	Berkshire	23·9	10·1
South Wales	33·1	14·6	Cheshire	27·5	12·7	Surrey	23·7	11·2
Northumberland ...	32·7	13·1	Lincolnshire	26·7	12·3	Hertford	23·6	9·2
Nottingham	31·6	14·7	North Wales	26·6	12·3	Oxford	23·5	10·9
Warwick	30·3	14·1	Shropshire	26·6	10·4	Devonshire	23·4	12·4
Derbyshire	30·3	12·7	Suffolk	25·8	10·7	Hereford	23·4	10·8
Essex	29·9	11·9	Northampton	25·7	11·2	Bedford	23·3	10·7
Lancashire	29·8	15·0	Buckingham	25·6	10·4	Cornwall	23·2	13·9
North Riding, Yorkshire	29·5	13·5	Gloucester	25·6	12·1	Somerset	23·0	10·0
East Riding ,,	29·2	12·9	Norfolk	25·5	12·8	Dorset	22·9	8·7
Middlesex	28·9	12·4	Kent	25·4	11·6	Sussex	22·1	9·8
Leicester	28·7	13·6	Hampshire	24·7	11·4	Westmoreland	21·9	8·2
West Riding, Yorkshire	28·5	14·4	Wiltshire	24·3	9·8	Rutland	21·9	7·1

Thus we see that notwithstanding a few irregularities birthrates and infantile mortalities are closely associated in the registration districts of England and Wales—the irregularities being probably due to the differing ages, conjugal habits, sex proportions and other conditions. Where these factors are alike I think the infantile mortality becomes the chief determinant of the birthrate. As we have seen from a previous table, although natural selection is responsible for a larger birthrate its further operation tends to restore the balance.

It might be inferred from the table showing the survivors at 5 years of age in various countries that by the operation of natural selection in countries where the mortality was heavy during that period the survivors could be regarded as specially selected lives, and that their subsequent expectation of life would be greater than in those countries where the mortality was comparatively light. But this is not always the case, as will be seen from the following table, which is extracted from life tables given by Mulhall from the experiences of the censuses of 1880 and 1881. I have added the results of New South Wales, which I have computed for the census of 1891.

Survivors at various ages of 1000 *born.*

Age	England	France	Prussia	Austria	Italy	Spain	Sweden	Norway	Belgium	Switzerland	New South Wales
20	706	685	616	532	554	496	711	760	699	689	798
40	597	566	514	423	462	412	621	644	589	578	696
60	405	408	351	267	328	292	473	494	419	393	500
80	96	104	65	44	65	69	139	161	101	69	116

From these figures may be derived the numbers of survivors to age 80 of every 1000 who attain age 5 and which are as follows:

England	126	Sweden	178
France	138	Norway	192
Prussia	95	Belgium	134
Austria	72	Switzerland	92
Italy	103	New South Wales	137
Spain	121		

From this it is seen that those countries with one exception—France—with heavy mortality under 5 years of age do not yield as favourable a result at age 80 even after such a severe weeding out, as those countries where the mortality under 5 is light.

E. *Reproductive Selection.*

Professor Pearson says* with regard to the question "Does Society ultimately recruit itself from above or below?"

"(*a*) The fertility of one class must be shown to be sensibly greater than that of another class. Now the causes of fertility for two classes in the same community differ comparatively slightly. Hence the small but sensible effect of natural selection must be carefully estimated and allowed for, i.e., we must deal with net fertilities.

"(*b*) But the net fertility of one class may still be greater than that of a second and yet the second increase at a greater rate. We have to determine how far the greater fertility is merely potential. We must ascertain what proportion of the two classes remain unmated.

"(*c*) Not only may the net fertility be greater—i.e., the number of children per family who survive infancy, say live to 15 years, be greater—but also the percentage of mating may be greater in one class than another and yet the same class may yet have no reproductive advantage. We ought further to take into account the *adult* death rates in the two classes. Natural selection may not only be effective in modifying the gross fertility into a net fertility but also as an adult death rate."

The "Vital Statistics" of New South Wales for the years 1895 to 1899 inclusive throw much light upon this question of reproductive selection. In them tables will be found showing the occupations of married men who died during that period together with the total offspring—distinguishing the living and dead. The ages of the living offspring are not given, so that the net fertility must be regarded as the number of offspring living at the time of the death of the father and irrespective of their ages. Thus the conditions contained in (*a*) and (*c*) above except that the net fertility is as just stated are fairly complied with, and as the

* *The Chances of Death*, Vol I. pp. 96 and 97.

results can be modified in accordance with the rates of marriage in the various classes of occupations, condition (*b*) will also be complied with.

The occupations are divided into six classes, viz., I. Professional (mainly engaged in the government and defence of the country, and in satisfying the moral, intellectual and social wants of its inhabitants). II. Domestic (persons engaged in the supply of board and lodging and in rendering personal service for which remuneration is usually paid). III. Commercial (persons directly connected with the hire, sale, transfer, distribution, storage, and security of property and materials). IV. Industrial (persons principally engaged in various works of utility, &c., i.e., artisans). V. Pastoral, Agricultural, and Mining. VI. Indefinite. Table XXVIII. shows the number of deaths in the various classes during the years 1895 to 1899, together with the living and dead offspring, the average to each class, and the proportion of living to total offspring.

TABLE XXVIII.

Occupation	Number of Deaths	Offspring			Average Offspring			Percentage of Living to Total Offspring
		Living	Dead	Total	Living	Dead	Total	
I. Professional	886	2896	972	3868	3·269	1·097	4·366	74·9
II. Domestic	707	2429	847	3276	3·436	1·198	4·634	74·1
III. Commercial	3116	10929	3706	14635	3·507	1·189	4·696	74·7
IV. Industrial	5074	19176	7143	26319	3·779	1·408	5·187	72·9
V. Pastoral, Agricultural and Mining	5564	26730	7484	34214	4·804	1·345	6·149	78·1
VI. Indefinite	454	1821	727	2548	4·011	1·601	5·612	71·5
Total	15801	63981	20879	84860	4·049	1·321	5·370	75·4

Taking the results as they stand and disregarding the Indefinite class (the members of which should really have been distributed amongst the other classes) we find that the essentially Rural class—Pastoral, Agricultural, and Mining—yields the largest gross and largest net results and the Professional the lowest in both. The order of gross fertility which is also that of net fertility is Pastoral &c., Industrial, Commercial, Domestic, Professional. We also see with regard to the last four classes which comprise for the most part the urban occupations that the greatest mortality occurs in the largest families, with the slight exception in the cases of the Domestic and Commercial classes. This is in agreement with the results obtained by Professor Pearson in dealing with Rubin and Westergaard's Copenhagen statistics. But in this instance natural selection has not disturbed the order of gross fertility. But the rural class with by far the greatest fertility has also the least infantile and juvenile mortality, and it would seem likely that this class would ultimately recruit society. But the results shown in the sixth column of Table XXVIII. have yet to be modified by the marriage rates of each

class, and fortunately I have been able to obtain these with a great degree of accuracy. The total number of males over 20 years of age in each class was obtained at the census of 1901, and the occupations of the bridegrooms have also been obtained for the years 1901 and 1902, and which are published in the "Vital Statistics" of New South Wales for those years. Hence the marriage rates per 1000 adults of each can readily be ascertained. The statistics and results are shown in Table XXIX.

TABLE XXIX.

Marriage Rates of Various Classes of Occupations.

Class of Occupation	Number of Adults (20 upwards) at Census 1901	Number of Marriages Mean of Years 1901 and 1902	Marriage Rate per 1000 Adults	Index No. Mean = 1
Professional	23300	764·5	32·81	1·181
Domestic	17277	272	15·72	·566
Commercial	91939	2759·5	29·99	1·079
Industrial	102071	4130·5	40·46	1·456
Agricultural, Pastoral, and Mining	139947	2545·5	18·18	·654
Indefinite	3575	39·5		
Total	378109	10511·5	27·79	

By applying the index number shown in the last column to the net fertility shown in Table XXVIII. we obtain finally as the comparative net fertility:

1st, Industrial	Class	5·502,
2nd, Professional	„	3·861,
3rd, Commercial	„	3·784,
4th, Pastoral, Agricultural, &c.	„	3·142,
5th, Domestic	„	1·945.

Thus although natural selection failed to change the position of the classes in the net fertility from those occupied in the gross yet the class marriage rates have completely done so. The rural class with the highest gross and net fertilities, with the least mortality amongst the offspring, and also as we shall presently see with the lowest adult mortality, now falls to nearly the lowest position, whilst the industrial class with both these mortalities the highest occupies the highest position through the frequency of mating in that class. We have seen that the average gross fertility of the agricultural and pastoral class is 6·149, and the industrial 5·187 —merely a difference of one child per father; but this difference is probably not altogether due to a difference in actual fertility but to the heavier adult mortality in the latter class and consequently shortened life. This will be seen from the following

statement showing the adult deaths per 1000 of each class, and which have been calculated in the same way as the marriage rates:

Class of Occupation	Mortality Rate per 1000 Males over 20 years
Agricultural, Pastoral, &c	12·69
Commercial	13·29
Professional	15·73
Domestic	16·59
Industrial	22·72
Total	16·47

Thus notwithstanding an adult mortality far in excess of the other occupations and with the highest infantile and juvenile mortalities the frequency of marriage is also so much in excess that as Professor Pearson puts it, "after all society recruits itself from below," that is from the artisans—although as his statistics referred only to the City of Copenhagen he was not in possession of information regarding the rural occupations. This class it is seen with the highest fertility and the lowest mortality at every age absolutely fails to contribute its fair quota to the community in consequence of its low marriage rate.

When we examine the gross fertilities of the various classes (remembering that these are the results of all durations of marriage and of widowers, as well as of married men whose wives survive them) and compare with the Anglo-Saxon and Danish statistics, we again see that up to the present there is but little Malthusian restraint upon the population in New South Wales—what little there may be apparently being confined to the Professional, Domestic, and Commercial classes. The ages of the brides at marriage in these classes have not been tabulated, but it may be of interest to show the average age at marriage of the bridegrooms, which according to the marriages of the years 1901 and 1902 were as follows:

Class	Average Age at Marriage
Professional	30·22 years
Domestic	30·97 „
Commercial	28·88 „
Industrial	28·57 „
Agricultural, Pastoral, &c.	29·18 „

As the classes of occupations are rather comprehensive I append Table XXX. which shows similar information to Table XXVIII. for some of the sub-classes.

The agricultural class is unquestionably the most prolific in the community accompanied by an exceedingly low mortality among its offspring. The medical profession would appear to be conspicuously below the average of the whole community, whilst ministers of religion just reach that average. The comparatively heavy mortality amongst the offspring of the artisan classes is significant.

TABLE XXX.

Occupation		Deaths	Total Offspring			Average Offspring			Percentage of Living to Total Offspring
Class	Sub-Class		Living	Dead	Total	Living	Dead	Total	
I.	Religion*	68	264	101	365	3·882	1·485	5·367	72·3
I.	Health†	151	475	148	623	3·146	·980	4·126	76·2
IV.	Houses and Buildings‡	1173	4614	1822	6436	3·934	1·553	5·487	71·7
IV.	Labourers, &c. ...	1488	5527	1933	7460	3·715	1·299	5·014	74·1
V.	Agricultural	3149	16613	4377	20990	5·275	1·390	6·665	79·1
V.	Pastoral	1024	4712	1249	5970	4·610	1·220	5·830	79·1

TABLE XXXI.

Distribution of Fertility in Occupations. Deaths of Married Men, New South Wales, 1895—1902.

Number of Children in Family	Class of Occupation											
	Professional		Domestic		Commercial		Industrial		Agricultural, Pastoral, &c.		All§	
	Deaths	Per cent.	Deaths	Per cent.	Deaths	Per cent.	Deaths	Per cent.	Deaths	Per cent.	Deaths	Per cent.
0	238	15·276	240	19·818	756	14·237	1136	12·900	899	10·103	3372	12·719
1	171	10·976	116	9·579	539	10·151	667	7·575	602	6·766	2151	8·113
2	161	10·334	110	9·083	529	9·962	819	9·300	606	6·811	2267	8·551
3	155	9·949	108	8·918	510	9·605	742	8·427	580	6·518	2163	8·159
4	138	8·858	92	7·597	501	9·435	734	8·335	648	7·283	2169	8·181
5	135	8·665	84	6·936	476	8·965	722	8·199	645	7·249	2103	7·932
6	110	7·060	94	7·762	384	7·232	720	8·176	631	7·091	1999	7·540
7	90	5·777	69	5·698	350	6·592	673	7·642	721	8·103	1953	7·366
8	96	6·162	70	5·781	336	6·328	666	7·563	750	8·429	1984	7·483
9	88	5·648	88	7·267	283	5·330	522	5·928	680	7·642	1720	6·488
10	73	4·685	41	3·386	212	3·992	497	5·644	684	7·687	1564	5·899
11	35	2·246	40	3·303	182	3·427	362	4·111	510	5·732	1159	4·372
12	22	1·412	22	1·817	105	1·977	235	2·669	421	4·731	830	3·131
13	19	1·220	20	1·652	71	1·335	144	1·635	228	2·562	488	1·841
14	11	·706	9	·743	34	·640	84	·954	139	1·562	278	1·049
15	6	·385	4	·330	13	·245	32	·363	61	·686	120	·452
16	2	·128	2	·165	14	·264	24	·273	49	·551	93	·351
17	3	·193	2	·165	8	·151	9	·102	10	·112	34	·128
18	2	·128	—	—	4	·075	8	·091	15	·169	30	·113
19	2	·128	—	—	1	·019	4	·046	11	·124	18	·068
20	1	·064	—	—	—	·000	1	·011	5	·056	7	·026
21	—	—	—	—	2	·038	3	·034	2	·022	7	·026
22	—	—	—	—	—	—	1	·011	—	—	1	·004
27	—	—	—	—	—	—	—	—	1	·011	1	·004
29	—	—	—	—	—	—	1	·011	—	—	1	·004
Totals	1558	100·000	1211	100·000	5310	100·000	8806	100·000	8898	100·000	26512	100·000

* Ministers of Religion, &c.
† Doctors, Chemists, &c.
‡ Carpenters, Bricklayers, Masons, Plumbers, &c.
§ Including the Indefinite class not shown in Table.

In Table XXVIII. the results were obtained from the deaths during 1895–9 when the tabulation distinguished between the living and dead offspring. Since that date the offspring have not been so distinguished, but only the total or gross shown. As it may be of some value to show these for the longer period 1895–1902 and to show the range of families in each class I have compiled Table XXXI.

The mean family, median family, median children and percentage of married men producing half the children in each class are shown in Table XXXII.

TABLE XXXII.

Occupation	Mean Family	Median Family	Median Children	Percentage producing 50 per cent. of Children
Professional	4·55	3·39	7·09	22·58
Domestic	4·52	3·34	7·43	23·12
Commercial	4·70	3·64	7·08	23·30
Industrial	5·26	4·42	7·57	25·11
Agricultural, Pastoral, and Mining	6·21	5·74	8·56	27·37
All	5·39	4·54	7·85	25·07

It will be noticed that the averages—with one slight exception—are sensibly greater in each class for the period 1895–1902 than for the period 1895–9. This is not due to increasing fertility during the additional period but to the fact that New South Wales is annually increasing its proportion of old people which has not yet reached that of a normal community—in consequence of its comparatively recent settlement. For instance, the proportions of the population over 60 years of age in New South Wales were at the censuses of 1861, 1871, 1881, 1891, and 1901, 3·1, 4·0, 4·3, 4·4, and 5·6 per cent. respectively, whilst the proportion in England and Wales at the last census was 7·4 per cent. But as this increase in age is common to all sections of the community the comparison between the fertilities of the various occupations is in no way vitiated.

Tables XXXI. and XXXII. need but little comment. It is seen that Professor Pearson's conclusion that generally 25 per cent. of the married contribute 50 per cent. of the next generation is corroborated, though the percentage varies in the various classes, being largest in the rural and smallest in the professional classes. Further that his view that society is recruited from below is also sustained—that it is the comparatively short-lived artisan class, notwithstanding the highest mortality among its offspring, that is contributing the largest quota to the population.

CONTRIBUTIONS TO THE STUDY OF INTERRACIAL CORRELATION.

By E. TSCHEPOURKOWSKY, Moscow.

Many authors have tried to explain mechanically several of the distinctive characters of the races of man, especially the different forms of the skull. I may mention Nystrom's hypothesis: he asserted that the head-form may be the product of the work of muscles on the one hand, and of the inner pressure of blood on the other. We have also similar theories due to Ranke and Thomson, but the evidence on which they are based is very slight. I propose in this paper to state some considerations based on biometrical methods. In a paper read before the Congress of Naturalists and Physicians assembled in St Petersburg in 1900 I made the following statements based on the measurements of 700 skulls*.

(1) The form of the skull is connected with the form of its base, and the variation of the first produces a series of the variations in the second, some of which resemble embryonic growth. (2) We do not find between the lengths and breadths of skulls in the same race a relation which is in accordance with the principle spoken of as that of "compensational growth." If one increases, the other also increases. But if we pass from one race to another we find that with the growth of length there is a decrease of the breadth, and *vice versa*. This arises from the fact that the most brachycephalic races have the greatest breadths and the smallest lengths. The effect would be the same if, instead of different races, we took dolichocephalic and brachycephalic groups in the same race. (3) As the decrease of the skull length is connected with the decrease of the length of skull-base and of the face-length, we should expect that lower races with powerful jaws and small cranial capacity must have a tendency to dolichocephaly, and again that the decrease of the facial parts and the development of the brain are influences which are favourable to brachycephaly. (4) In the geographical distribution of the cephalic index we have two well-marked regions: generally speaking southern continents are dolichocephalic and northern are brachycephalic. We can perhaps distinguish on the earth two

* Extracted from the Report of the Congress.

centres isolated by the highest mountains of the globe, from which only the northern centre was favourable to the development of brachycephaly. The southern remained in a primitive state.

In the same report and in that read by me before the Anthropological Congress in Worms I made the following deductions, based on the measurements of 1,300 women and their new-born children*: (1) that the head-form of the mother is inherited by the new-born child; and (2) that not only is the form of the skull, but also the form of the base determined in this very early period of life. As the base is then relatively very small we can hardly speak about its influence on the form of the skull: both are equally inherited.

These last facts brought me more and more to the conclusion that sexual selection has played an important *rôle* in the variation of the cephalic index. Ripley denies this on the ground that it is not a prominent feature of the human body. But selection may be at work if the index be simply correlated with selected characters. In order to ascertain this, and throw light on many other problems which I hope shortly to treat in a special anthropological essay, I have collected material for the study of interracial correlation. This material is partly extracted from Ivanovsky's *Anthropological Constituents of the Population of Russia*, partly from Deniker's *Races of Man*, and as far as regards crania from different authors and from my own measurements. From the data given below, upon which the cross correlations can also be studied, the correlation tables (Appendix, Tables 1 to 28) were formed and the constants calculated, which are represented in the following table. Many conclusions can be drawn from this. I will mention some of the results.

1. About the correlation between stature and head form there exist the most contradictory opinions between anthropologists. Interracially for the races of Russia I found a very sensible *negative* correlation. The control table for other races of the earth was formed on Deniker's data, all Russian races of the preceding correlation table being excluded, and I obtained the same *negative* result. As the total number of races in both tables is 176, this result is most probably correct. I had very few data for women—only 28 cases—and they give me a very slight positive correlation. The material is not large enough to draw conclusions concerning the sexual difference. It follows from the masculine correlation that the head-form changes when stature is selected.

2. The correlation between the cephalic index and the shape of face, i.e. the relation of bizygomatic breadth to the total height of the face, measured from the top of the forehead to the chin, is positive and very sensible. It proves that the head-form can be influenced by sexual selection, because the shape of face is a prominent feature of racial "habitus." But as we pass to the upper face index, i.e. to the relation upper face height to bizygomatic breadth, which is calculated

* "Ueber die Vererbung der Kopfform von Seiten der Mutter," *Correspondenzblatt der Deutschen Anthrop. Ges.* No. 12, 1903.

TABLE I. *Interracial Correlation and Variation.*

Tables		No. of Cases	First Character A	Second Character B	Standard Deviation A	Standard Deviation B	Mean A	Mean B	Correlation
1	Living Races	92	Stature	Cephalic Index	3·33	2·91	164·74	82·36	− ·184
2	" " (from Deniker)	84	"	" "	5·59	4·02	163·96	78·70	− ·224
3	" " " "	54	Cephalic Index ...	Face Index	2·97	2·56	82·87	77·59	+ ·262
4	Crania	47	" " ...	Upper Face Index ...	4·90	1·99	77·21	51·36	+ ·177
5	Living Races	65	" " ...	Nasal Index	3·10	6·73	82·78	64·35	+ ·050
6	" "	83	Stature	Head Length	3·43	2·77	164·98	185·89	+ ·096
7	" "	83	"	Head Breadth	3·43	4·50	164·98	153·22	− ·042
8	" "	53	"	Head Height	3·29	6·48	164·91	130·98	+ ·096
9	" "	58	"	Bizygomatic Breadth ...	3·01	4·84	164·83	142·33	− ·172
10	" "	50	"	Face Index	3·18	2·97	165·2	78·59	− ·164
11	Crania	43	Length	Breadth	4·68	6·49	182·12	137·65	− ·321
12	"	43	"	Height	4·73	4·49	181·93	135·86	+ ·615
13	"	43	Breadth	"	6·49	4·49	137·65	135·86	− ·132
14	"	32	Upper Face Height	Bizygomatic Breadth ...	3·60	4·28	68·03	131·66	+ ·641
15	"	32	Length	Upper Face Height ...	5·09	3·60	181·34	68·03	+ ·257
16	"	32	Breadth	" " ...	6·59	3·60	137·62	68·03	+ ·504
17	"	32	Height	" " ...	4·43	3·60	135·06	68·03	+ ·074
18	"	41	Length	Bizygomatic Breadth ...	4·89	4·12	182·19	131·95	+ ·489
19	"	41	Breadth	" " ...	6·07	4·12	137·15	131·95	+ ·257
20	"	41	Height	" " ...	4·49	4·12	136·08	131·95	+ ·087
21	"	38	Length	Minimum Frontal Breadth	5·13	2·63	182·21	95·21	+ ·320
22	"	38	Breadth	" " "	5·76	2·63	137·26	95·21	+ ·328
23	"	38	Height	" " "	4·61	2·63	136·06	95·21	− ·135
24	"	31	Skull Basis ...	Upper Face Height ...	3·34	3·73	100·55	68·22	+ ·124
25	"	42	" ...	Breadth	3·33	6·51	101·17	137·8	− ·262
26	"	40	" ...	Bizygomatic Breadth ...	3·34	4·15	101·30	132·02	+ ·255
27	"	42	" ...	Height	3·45	4·53	101·14	135·9	+ ·507
28	"	42	" ...	Length	3·34	4·90	101·14	182·31	+ ·688

Face Index on the living = 100 bizygomatic breadth/height from chin to top of forehead.
Upper Face Index on crania = 100 bizygomatic breadth/Upper face height.
Upper Face Height = distance from nasion to alveolar point.
Skull Basis = distance from nasion to basion.
Length and Breadth alone refer to maximum cranial length and breadth and Height alone to basio-bregmatic height.

mostly from my own measurements, we find a very interesting fact: the correlation of the cephalic index and upper face index is positive, i.e. the upper face height increases more rapidly with brachycephaly than the bizygomatic breadth, so that mongoloid races having very prominent jaws have, however, the upper face longer than negroid races. It depends, I think, upon the diminution of prognatism, and represents morphologically a more advanced state of phylogenetic growth. Perhaps it stands in connection with the more advanced development of the base in the case of brachycephals, which was first noticed by Schäffer. Intraracially I obtained the results represented in the Table II.

TABLE II.

Skulls ♂. *Intraracial Variation and Correlation of Cephalic Index and Upper Face Form.*

Race	No. of Cases	Means		Standard Deviations		Coefficient of Correlation
		Cephalic Index	Upper Face Index	Cephalic Index	Upper Face Index	
Australians	66	71·03	49·35	2·85	2·93	+·089
Negroes	77	74·34	51·27	3·60	3·95	+·182
Duke of York Islanders	53	70·90	48·70	2·91	3·22	−·093
Malays	60	80·20	50·23	4·70	2·80	−·185
Fijians	32	69·59	49·47	4·58	3·31	+·217
Papuans	39	72·36	49·51	4·41	3·40	−·255
Polynesians	44	77·68	50·23	5·20	3·70	+·002
Alfourous	19	80·73	50·89	5·71	3·82	−·302
Micronesians	32	74·41	51·15	3·69	3·71	−·251
Copts	34	77·44	54·53	4·83	3·10	−·147
Etruscans	47	78·15	54·15	2·93	2·75	−·021
Europeans	80	80·24	53·95	3·80	3·44	−·198
Ancient Thebans ...	152	75·05	53·52	3·17	3·54	−·067

It follows from them that intraracially the correlation is mostly negative.

For nasal and cephalic indexes I found a slight positive correlation for the races of Russia.

3. The interracial correlation of the diameters of the head with stature shows that the height—measured from tragus to vertex—and the length slightly increase, but the breadth decreases. In accordance with these correlations we find that with the increase of stature not only the breadth of skull, but also the bizygomatic breadth of the face decreases. It is perhaps a general rule that horizontal diameters of the head decrease with increase of stature while vertical and longitudinal diameters increase.

4. Passing now to the cranial correlations we find interracially the negative correlation between length and breadth, which is in accordance with Dr Alice Lee's researches. The correlation of length with height is large and positive, of height

with breadth negative. In the diameters of the face we find a high correlation between upper face height and bizygomatic breadth. Very interesting is the fact that the upper face height is more closely correlated with breadth than with length. The bizygomatic breadth on the contrary is more correlated with length than with breadth. Both diameters of the face are very slightly correlated with the height. The minimum frontal breadth is positively correlated with length and breadth and negatively with height.

It follows from the values of the above constants that interracially with the increase of the length of the skull the breadth diminishes; the height and basal length on the contrary increase as well as the stature.

5. For interracial variation we have the chief results presented in the Table III. below. We conclude from these constants that interracially the length of the

TABLE III.

Coefficients of Interracial Variation.

Character	Coefficients of Variation	
	Living Subjects	Skulls
Stature	2·02	—
Cephalic Index	3·58	6·34
Face Index	3·29	—
Upper Face Index ...	—	3·87
Nasal Index	10·40	—
Cephalic Length	1·49	2·57
Cephalic Breadth	2·93	4·71
Cephalic Height	4·95	3·30
Bizygomatic Breadth ...	3·40	3·14
Minimum Frontal Breadth	—	2·76
Upper Face Height ...	—	5·29
Skull Basis	—	3·32

skull is less variable than the breadth. The height in the case of skulls is also less variable than the breadth. The variation constants for length and breadth of living subjects are in accordance with those for the skulls, but the height is not to be compared in the two cases: in living subjects it is measured as the distance between tragus and vertex, and its great variability is, I think, partly owing to the impossibility of perfect measurement, partly to inconstancy of tragus. Very interesting also is the great variability of the upper face height as compared with that of the bizygomatic breadth.

From the above given constants we can deduce, I think, the following statement: *The varieties of the human head-form are mostly owing to the variation of the breadth.*

In concluding this study I wish to state some theoretical considerations which, although perhaps far from the absolute truth may, I hope, yet show how important for anthropology the biometrician's work will be in the future when aided by extended anatomical studies. I call first the reader's attention to the fact that the several diameters of the head in new-born children do not stand in equal relation to those of adults. In other words they need not accomplish the same growth to reach the adult stage. The most complete table of these differences will be found in the work of Papillaut. From my own measurements of the base of the skulls of new-born infants I have reached, as a general result, the conclusion that the breadths of the skull and of its base are far less developed in the embryonic stage than the lengths when we compare their ratios with those of adults. I give here some numerical data extracted from the work of Papillaut.

Character	Upper Face Height	Bizygomatic Breadth	Breadth of Skull	Length of Skull	Minimum Frontal Breadth
Relation : $\frac{\text{New-Born}}{\text{Adult}}$	44°/₀	48°/₀	56·5°/₀	60°/₀	60°/₀
Coefficient of Interracial Variation	5·29	3·14	4·71	2·57	2·76

Compared with the coefficients of interracial variation they show parallelism with only one exception—that of bizygomatic breadth. I cannot affirm that racial differences in the skull are simply differences of growth, but the above-mentioned parallelism seems to me not without interest when compared with the other point which is also connected with growth. We have seen that interracially and partly intraracially—probably as far as we can eliminate the influence of mixture—the cephalic index diminishes with stature, because the length and height increase and the breadth decreases. I have also shown that the length of the basis is *positively* related with the length and height and *negatively* with the breadth of the skull. Now the length of the basis consists of two parts, which are very different embryologically. The distance between sutura spheno-basilaris and the basion is the end of the primordial skeleton-axis, i.e. of the chorda dorsualis. The second part, i.e. the distance between nasion and sutura, is connected with the development of the brain and partly influenced by sinus frontales. Manouvrier has shown that only the first part of the base-length depends on stature. It has much more to grow in order to reach the adult stage than the second, which is developed very early. The reason is the great development of the embryonic brain owing perhaps to its physiological faculties. It follows from these considerations that the negative correlation between stature and cephalic index can be explained by the growth of the length from basion to sutura basio-sphenoidalis, which increases much more rapidly with the stature than others do because it is a part of the primordial body-skeleton. With the increase of this diameter the length of the skull and the basio-bregmatic height increase also, but the breadth compensationally decreases. As it follows from the tables, all these correlations are

observed only as averages, and separate races may present many exceptions. The cause of the negative correlation between stature and cephalic index is thus only one of many factors which can influence the head-form. We must, therefore, conclude that the diameters of the head and the cephalic index respectively depend on many causes. The anthropometric measurements are often artificial simplifications, and will, with the progress of anthropology, be made to approach more closely to those regions which are physiologically and embryologically individualized. The organic influences which exist in the individual, or within the limits of a race, can be masked, and correlations between numerical values of measurements inverted as we pass to interracial correlation because other factors come into play. The one of these factors is sexual selection, which is very powerful in the species *Homo*. Admit for instance that the general growth of an organism produces an elongation of the head. Then if the selection is connected with stature, and tall individuals survive, the new race thus formed must be long-headed. But if round-faced individuals become a type of racial beauty, the sexual selection works in the opposite direction, i.e. produces a round-headed tall race, so that in a particular case the organic influence can be masked.

The progress of the evolution of a race is the result of a number of organic correlations, and intellectual or instinctive impulses play an important part in it. Many correlations which are preserved interracially are organic, i.e. physiological correlations. Perhaps we may define the characteristics of a given race as its deviations from the most probable values, calculated from the coefficients of organic correlation. These deviations are the result of processes which were not the same in the evolution of all races, and which have been quite special for each individual race.

APPENDIX I. RACIAL MEAN CHARACTERS.

TABLE I. *Races of or adjacent to the Russian Empire.**

Race or Group	Authority	Head Index	Stature	Head Length	Head Breadth	Height / Length	Height	Length of Face	Facial Index	Bizygomatic Breadth	Nasal Index	Capacity	Body Length (rel.)	Relative Capacity
Abazeck (Caucasian Tribe) ...	Erkert	81·6 (11)	—	192 (11)	157	67·6 (11)	130	—	77·4	—	63·4 (11)	1466	—	—
Abazin (id.) ...	Sommier ...	82·5 (23)	1680 (23)	192 (23)	158	—	—	—	—	—	—	—	—	—
Afghans	Matziewsky ...	75·6 (18)	1681 (18)	189 (18)	143	74·4	149	190	—	—	50·9 (18)	1336	36·0 (18)	80
„	Maslowsky ...	84·0 (18)	1638 (18)	—	—	—	—	—	73·2	—	—	—	—	—
Ainos	Koganei	77·3 (45)	1567 (91)	194 (35)	150	64·6	125	185	78·1	145	68·7 (87)	1383	—	88
Aisoren (Baku)	Arutinow ...	85·8 (12)	1562 (10)	184 (12)	160	75·3 (10)	139	182 (12)	78·7	143	60·0 (12)	1518	35·7 (10)	97
„	Pantuchow ...	87·9 (45)	1671 (43)	177 (45)	155	—	—	—	—	—	64·7 (34)	—	—	—
Arabs	Maslowsky ...	82·0 (17)	1637 (17)	183 (17)	150	—	—	189	72·5	137	—	—	—	—
Arbunsumun (Kulga, Kalmuks)	Maziewsky ...	83·0 (50)	1611 (30)	186 (30)	154	—	—	184 (30)	—	—	56·9 (50)	—	36·4 (30)	—
Armenians	Ivanowsky ...	87·0 (160)	1672 (160)	181 (160)	157	72·8	132	184 (160)	78·3 (160)	144	63·5 (160)	1557	35·9 (160)	93
Bashkirs	Nasarow	83·2 (163)	1647 (163)	182 (163)	152	67·3 (193)	122	186 (160)	76·7 (160)	143	75·8 (160)	1308	30·4 (49)	79
Buriats	Talko-Grinzew ...	85·7 (535)	1631 (544)	187 (535)	160	—	—	192 (505)	79·9 (294)	153	57·0 (295)	—	35·7 (273)	—
Bow	Diff. Authorities	85·9 (816)	1631 (825)	186 (815)	160	69·5 (100)	129	189 (484)	79·7 (484)	151	62·1 (556)	1443	35·7 (343)	88
White Russians, Smolensk ...	Eichhold ...	81·3 (100)	1652 (100)	186 (100)	151	69·2 (100)	129	—	—	—	69·3 (100)	1380	—	84
„ Minsk ...	Rosdestwensky ...	—	1648 (57)	185 (57)	—	60·5 (50)	—	183 (57)	75·9 (57)	139	68·0 (57)	—	34·6 (54)	—
„ Elnin District	Shadrowizky ...	80·2 (445)	1675 (445)	190 (445)	152	—	—	187 (445)	76·3 (445)	143	69·4 (445)	—	—	—
Great Russians, Twer	Galaï	83·0 (200)	1660 (200)	179 (200)	149	70·6	126	183 (200)	74·6 (200)	137	69·7 (200)	1304	35·1 (200)	78
„ Jaroslaw ...	Wilga	81·8 (100)	1689 (100)	187 (100)	153	—	—	—	77·7 (100)	—	—	—	—	—
„ Riazan ...	Worobiew ...	81·5 (325)	1651 (325)	188 (325)	153	—	—	183 (323)	77·1 (323)	141	—	—	—	—
„ Kursk ...	Lukine	81·7 (235)	1640 (235)	185 (235)	151	68·8 (235)	127	180 (235)	77·5 (235)	140	66·7 (235)	1358	34·7 (235)	83
„ Toula ...	Grigoriew ...	83·6 (63)	1643 (63)	184 (63)	154	71·1 (63)	131	182 (63)	76·6 (63)	139	71·8 (63)	1406	35·2 (63)	86
„ Eriwan ...	Iwanowsky ...	82·3 (60)	1654 (60)	185 (60)	152	71·0 (60)	131	183 (60)	76·4 (60)	141	68·2 (60)	1398	35·0 (60)	85
„ Siemiriechansk	Zealand	81·3 (51)	1671 (51)	187 (51)	152	71·6 (51)	134	—	—	—	—	1435	—	86
„ Transbaïkalia	T. Grinzewich ...	83·1 (225)	1669 (175)	—	—	—	—	—	—	—	—	—	—	—
Votjaks	Maliew	81·9 (120)	1619 (119)	183 (120)	150	—	—	177 (188)	78·7 (100)	139	—	—	—	—
Gruzins	Chantre	85·4 (118)	1646 (232)	182 (80)	155	—	—	181 (40)	79·9 (40)	145	63·4 (40)	—	—	—
Dungans	Maziewsky ...	76·0 (50)	1659 (30)	188 (30)	143	71·9 (30)	135	199 (30)	—	—	51·8 (30)	1382	36·3 (30)	83
Jews, Warsaw	Elkind	81·9 (200)	1610 (200)	184 (200)	151	67·1 (200)	124	184 (187)	73·7 (147)	136	62·0 (147)	1328	33·9 (100)	83
„ Kiew	„	82·9 (438)	1625 (438)	—	—	—	—	—	—	—	—	—	—	—
„ Lithuania	„	81·7 (69)	1619 (68)	—	—	—	—	—	—	—	—	—	—	—
„ Kowno	Blechman ...	83·0 (52)	1623 (52)	188 (52)	156	—	—	179 (52)	76·8 (52)	137	—	—	—	—
„ Minsk	Stieda	82·2 (67)	—	187 (67)	154	62·2 (67)	116	181 (67)	—	—	—	—	—	—
„ Mohilew	Jakowenko ...	81·7 (186)	1617 (139)	183 (139)	150	—	—	181 (186)	76·2 (148)	138	59·9 (170)	—	—	—
„ South Russia	Weissenberg ...	82·4 (100)	1648 (259)	183 (100)	151	66·1 (100)	121	—	—	—	—	1299	—	79
„ Odessa	Iwanowsky ...	82·8 (50)	1626 (50)	183 (50)	151	66·7 (50)	122	183 (50)	76·1 (50)	139	62·4	1307	34·0 (50)	80
„ Transcaucasia	Pantuchow ...	87·5 (43)	—	184 (43)	143	—	—	184 (11)	76·9 (11)	142	62·7 (11)	—	—	—

* The numbers in brackets refer to the number of individuals in the series.

TABLE I.—(*continued*).

Race or Group	Authority	Head Index	Stature	Head Length	Head Breadth	Height / Length	Height	Length of Face	Facial Index	Bizygomatic Breadth	Nasal Index	Capacity	Body Length (rel.)	Relative Capacity
Eside	Horoshanko ...	77·9 (33)	1633 (24)	140 (33)	148	74·2 (29)	141	—	—	—	61·6	1480	—	91
Zirian	Nalimow ...	87·2 (98)	1628 (97)	177 (98)	154	76·6 (96)	136	192 (98)	76·4 (98)	147	64·7 (98)	1406	36·6 (70)	86
Ingoushi	Pantuchow ...	82·3 (55)	1713 (60)	190 (54)	157	—	—	186 (25)	78·4 (25)	146	67·9 (23)	—	—	—
Istigarzin	Danilow	80·4 (31)	1652 (31)	184 (31)	148	74·7 (31)	137	186 (31)	74·6 (31)	139	—	1370	36·8	83
Kabards	Wishgorod ...	83·7 (40)	1678 (40)	186 (40)	156	72·9 (40)	136	178 (39)	80·5 (37)	143	65·0 (37)	1475	—	87
Kalmuks, Astrakan	Korolew	81·1 (91)	1643 (93)	189 (90)	153	64·2 (92)	121	185 (95)	79·3 (95)	147	73·4 (97)	1343	35·5 (91)	82
„ „	Diff. Authorities	82·0 (212)	1643 (236)	189 (212)	155	62	—	—	—	—	—	—	—	—
„ Chackari	Maziewsky ...	83·6 (30)	1622 (30)	186 (30)	156	69·8 (30)	130	184 (30)	—	—	54·8 (30)	1424	35·7 (30)	88
„ Tourfan	„ ...	85·8 (30)	1631 (30)	180 (30)	155	70·5 (30)	127	186 (30)	—	—	58·5 (30)	1357	34·4 (30)	83
Karaims	Weissenberg ...	84·8 (20)	1645 (20)	184 (20)	156	—	—	—	—	—	62·2	—	—	—
Kara-Kirghiz	Zealand	87·3 (40)	1653 (40)	184 (40)	161	72·0 (40)	132	183 (40)	—	—	76·2 (40)	1464	36·0 (40)	88
Karachai	Sommier ...	84·4 (21)	1675 (20)	191 (21)	162	—	—	—	—	—	—	—	—	—
Karelians	Retzius	81·2 (27)	1720 (28)	186 (27)	151	—	—	189 (27)	73·7 (27)	139	—	—	—	—
Kirghiz, Boukieew-orda ...	Charusin ...	86·3 (145)	1629 (141)	187 (139)	162	—	—	184 (95)	—	—	—	—	—	—
„ Sredni orda	Iwanowsky ...	89·4 (99)	1651 (99)	181 (99)	163	72·6 (99)	131	182 (99)	80·4 (99)	148	71·8 (99)	1451	36·1 (99)	88
„ Sir-Daria	Maslowsky ...	87·9 (54)	1645 (54)	183 (54)	161	—	—	—	—	—	—	—	—	—
„ Kouldsa	Maziewsky ...	83·2 (30)	1631 (30)	183 (30)	152	75·9 (30)	139	—	—	—	52·4 (30)	1452	36·0 (30)	89
Chinese Koulsa	Maziewsky ...	78·4 (30)	1670 (30)	186 (30)	146	77·1 (30)	144	192	—	—	53·4 (30)	1465	35·8 (30)	87
Koumik	Swidersky ...	87·0 (107)	1644 (128)	181 (107)	157	71·4 (104)	129	181 (53)	79·5 (53)	144	65·3 (107)	1393	—	85
Kurds, Transcaucasia ...	Iwanowsky ...	77·6 (300)	1686 (300)	189 (300)	147	70·1 (300)	133	183 (300)	75·2 (300)	138	62·3 (300)	1401	35·4 (300)	83
„ Turkomans	Chantre	78·5 (140)	1677 (60)	193 (140)	152	—	—	—	—	—	67·7 (140)	—	—	—
„ Syrians	„	82·5 (35)	—	193 (35)	159	—	—	—	—	—	72·8 (35)	—	—	—
„ Persians	Danilow	77·7 (10)	1642 (19)	189 (19)	147	—	—	—	—	—	—	—	36·2 (18)	—
Laze	Chantre	87·5 (27)	—	192 (27)	163	—	—	—	76·1 (27)	—	64·4 (27)	—	—	—
Lettish	Waeber	80·5 (60)	1705 (60)	190 (60)	153	60·4 (60)	115	185 (60)	—	—	63·8 (60)	1297	—	76
Lezgin	Courdow	86·7 (89)	1680 (89)	183 (89)	159	75·8 (89)	139	177 (89)	81·9 (89)	145	—	1504	36·7 (89)	89
Livonians	Waldhauer ...	79·9 (100)	1736 (100)	190 (100)	152	62·1 (100)	118	184 (100)	—	—	—	—	—	—
Lithuanians	Brennsohn ...	81·8 (60)	1662 (60)	191 (60)	156	61·2 (60)	117	183 (60)	—	—	72·9 (60)	1316	—	79
„	Baronass ...	82·7 (70)	1656 (70)	184 (70)	152	—	—	165 (70)	75·2 (70)	139	—	—	—	—
Lopar	Sommier ...	84·0 (24)	1548 (24)	183 (24)	154	66·7 (22)	122	181 (25)	—	—	—	1326	37·9 (20)	86
Little Russians, Kiew ...	Diebold	84·5 (200)	1669 (200)	184 (200)	155	71·4 (200)	131	179 (200)	—	—	64·5 (200)	1373	—	82
„ Charkow ...	Krasnow ...	83·2 (80)	—	—	—	—	—	176 (80)	80·4 (80)	142	69·3 (80)	—	—	—
„ Charnigow ...	Bielodied ...	85·6 (120)	1666 (120)	182 (120)	156	—	—	181 (120)	77·8 (120)	141	71·6 (120)	—	—	—
„ Woroniege ...	Kosuehow ...	71·1 (102)	1632 (102)	187 (102)	152	—	—	187 (102)	77·2 (102)	144	—	—	—	—
„ Wolin ...	„ ...	79·8 (48)	1657 (48)	186 (48)	149	—	—	182 (48)	75·2 (48)	137	66·8 (48)	—	—	—
„ Kouban ...	Gilchanko ...	82·1 (61)	1701 (61)	185 (61)	152	—	—	177 (61)	79·6 (61)	141	69·6	—	—	—
Mesleganz	Danilow	76·5 (22)	1643 (22)	190 (22)	145	70·3 (22)	133	179 (22)	74·7 (22)	134	—	1392	36·4 (22)	85
Meszeriaks	Zograf	81·2 (44)	1655 (45)	189 (44)	154	77·1 (44)	146	189 (44)	—	—	63·5 (15)	1518	—	92
Mingrelians	Pantuchow ...	83·2 (12)	1647 (15)	197 (12)	162	66·0 (10)	130	178 (15)	78·6 (15)	140	59·1 (15)	1545	—	93

TABLE I.—(*continued*).

Race or Group	Authority	Head Index	Stature	Head Length	Head Breadth	Height / Length	Height	Length of Face	Facial Index	Bizygomatic Breadth	Nasal Index	Capacity	Body Length (rel.)	Relative Capacity
Moravia	Maïnow	83·2 (162)	1643 (162)	187 (162)	156	—	—	189 (162)	—	—	66·7 (162)	—	—	—
Orochons	Margaritow ...	82·8 (37)	1545 (37)	184 (37)	152	—	—	—	—	—	71 (37)	—	—	—
Ossetes...	Riskine	81·5 (300)	1690 (300)	190 (300)	155	71·9 (120)	136	178 (120)	81·3 (120)	145	66·3 (300)	1492	33·6 (120)	88
Ostiaks	Chugunow ...	79·3 (100)	1595 (100)	192 (100)	152	71·0 (58)	136	181 (100)	79·6 (100)	144	74·1 (100)	1490	—	94
Permiaks	Maliew	82·4 (100)	1618 (100)	180 (100)	148	—	—	173 (100)	81·0 (100)	140	—	—	—	—
Persians	Diff. Authorities	78·7 (123)	1677 (142)	189 (123)	149	71·5 (62)	135	188 (111)	74·3 (105)	140	—	1432	—	85
„	Danilow	78·4 (46)	1654 (46)	188 (46)	147	72·2 (46)	136	185 (45)	73·9 (45)	137	66·8 (14)	1423	36·1 (45)	86
Poles, Warsaw	Elkind	80·8 (226)	1640 (191)	185 (204)	150	—	—	179 (207)	76·8 (207)	138	63·3 (226)	—	34·3 (223)	—
„ Lublin	Dzerzinsky ...	82·7 (116)	1655 (116)	186 (116)	154	—	—	186 (100)	75·3 (100)	140	72·3 (100)	—	34·7 (100)	—
„ Radom	Olecknowiz ...	83·0 (131)	1617 (131)	186 (131)	155	—	—	—	—	—	—	—	—	—
Samoyedi	Zograf	82·5 (22)	1579 (22)	185 (22)	153	72·1 (20)	134	192 (21)	—	—	—	1430	38·8 (22)	90
„	Sommier ...	84·4 (60)	1540 (62)	189 (68)	160	—	—	—	—	—	—	—	—	—
Sartes	Matziewsky ...	83·9 (30)	1696 (30)	179 (30)	150	76·1 (30)	137	185 (30)	—	—	54·4 (30)	1398.	35·4 (30)	82
Sibo-Shibins	„ ...	82·4 (30)	1668 (30)	184 (30)	152	74·2 (30)	137	199 (30)	—	—	51·8 (30)	1442	35·3 (30)	86
Tajiks	Maslowsky ...	84·8 (356)	1691 (360)	184 (356)	156	—	—	193 (356)	73·3 (356)	142	—	—	—	—
Taranchi	Païssel	86·9 (300)	1646 (307)	179 (300)	155	72·1 (360)	129	185 (100)	77·3 (300)	143	66·5 (300)	1328	36·1 (60)	81
Tatars, Kazan	Warushkine ...	82·1 (206)	1645 (206)	186 (207)	153	68·1 (204)	127	179 (207)	78·6 (205)	141	70·7 (199)	1377	—	84
„ Kasim	Bensenger ...	82·8 (30)	1642 (30)	184 (30)	152	69·3 (30)	128	—	—	—	63·9 (30)	1368	—	83
„ Ufa	Talko-Grinzewich	80·9 (61)	1620 (61)	185 (61)	149	—	—	—	—	—	—	—	—	—
„ Crimea	Diff. Authorities	74·5 (107)	1661 (280)	186 (85)	157	—	—	177 (85)	—	—	—	—	35·3 (50)	—
„ „	Lijin	80·6 (50)	1665 (50)	188 (50)	152	—	—	174 (50)	77·0 (50)	134	—	—	—	—
„ Azerbeïjan	Diff. Authorities	77·6 (207)	1685 (172)	186 (207)	144	70·9 (631)	132	184 (75)	75·9 (72)	140	63·2 (169)	1355	36·5 (34)	80
Tats	Chantre	78·6 (38)	1701 (21)	188 (38)	148	—	—	—	—	—	67·6 (38)	—	—	—
Telenguts	Louzenko ...	86·1 (210)	1636 (209)	186 (210)	160	69·9 (180)	130	188 (210)	80·3 (210)	151	69·1 (210)	1452	34·3 (174)	88
Torguts, Tarbagatai	Iwanowsky ...	84·5 (73)	1633 (178)	188 (73)	159	69·3 (73)	130	186 (73)	85·2 (73)	159	60·5 (73)	1452	35·0 (42)	89
„ Koulja	Maziewsky ...	84·5 (30)	1623 (30)	186 (30)	157	71·3 (30)	133	187 (30)	—	—	57·5 (30)	1448	35·0 (30)	89
Tungus...	Maïnow	82·3 (87)	1631 (86)	192 (86)	158	65·2 (80)	125	185 (86)	81·5 (85)	151	56·9 (99)	1428	35·9 (21)	88
Turks	Pantuckow ...	84·7 (40)	1660 (40)	183 (40)	155	—	—	—	—	—	56·5 (13)	—	—	—
Turkomans	Iaworsky ...	75·6 (58)	1696 (58)	193 (58)	146	68·6 (58)	132	185 (58)	—	—	—	1408	—	83
Oudins	Arutinow ...	86·9 (150)	1644 (149)	183 (150)	160	73·1 (149)	134	181 (150)	79·6 (150)	144	57·7	1460	33·0 (149)	89
Uzbegs	Maslowsky ...	86·6 (196)	1678 (196)	182 (196)	158	—	—	190 (196)	77·4 (196)	146	—	—	—	—
Finns, Tawastehus	Retzius	82·3 (24)	1679 (22)	186 (24)	153	—	—	176 (24)	79·4 (24)	140	—	—	—	—
„ Wiborg	Kolmogorow ...	80·4 (150)	1666 (150)	190 (150)	153	—	—	183 (150)	76·8 (150)	140	81·2 (150)	—	—	—
Chalchas	Talko-Grinzewich	81·9 (30)	1611 (36)	189 (136)	155	—	—	197 (35)	76·6 (35)	151	57·1 (35)	—	36·4 (35)	—
Cheremiss	Maliew	79·0 (49)	1615 (48)	184 (48)	145	—	—	—	—	—	—	—	—	—
Chukchi	Olsufiew... ...	81·8 (14)	1660 (14)	187 (14)	153	71·9 (14)	134	—	—	—	—	1439	—	87
Esthonians	Grube	79·3 (100)	1643 (100)	194 (100)	154	—	—	187 (100)	—	—	—	—	—	—
Jakuts	Maïnow	82·3 (207)	1624 (207)	191 (207)	157	69·1 (207)	132	187 (207)	79·2 (207)	148	—	1478	—	91
„	Witachewsky ...	82·3 (46)	1607 (46)	190 (46)	156	70·5 (44)	134	193 (46)	—	—	—	1473	—	91

TABLE II.

Living Races from Deniker's Races of Mankind.

Race or Group	Stature	Head Index	Race or Group	Stature	Head Index
Aetas	146	85	Korumbas	156	77
Andamanese	148	81	Kulu-Lahulis	161	75
Annamese	157	83	Laotians	159	84
Arabs	166	76	Lapps	153	87
Australians	167	74	Malé	156	75
Badagas	166	72	Magyars	165	84
Baluchi	166	80	Mal-Paharias	158	76
Baltis	161	74	Mandingans	170	75
Batekes	164	74	Maricopas	172	83
Battas	160	81	Moquis	163	85
Bashilange	168	77	Mushikongos	166	72
Belgians	165	82	Nicobars	163	80
Bhumi (Bhils) ...	159	75	Omahas	173	82
Bilkula Indians ...	166	84	Pawnees	171	80
New Britain	162	77	Piedmontese	165	86
Burmese	165	83	Pimas	170	78
Caribs	157	80	Polynesians	173	80
Chakamas	160	84	Tahitians	173	85
Southern Chinese ...	161	81	Punjabis	168	74
Comanches	168	85	Rotti Islanders ...	160	77
Corsicans	163	77	Sardinians	162	77
Crow Indians	173	80	Santals	161	76
Dalmatians	171	87	Sicilians	163	79
Danakils	167	74	Singhalese	162	79
Dards	161	76	Siouans	173	80
Dravidians	163	74	Solorese	158	83
Eskimos, Alaska ...	163	79	Solomon Islanders ...	162	77
„ Greenland ...	162	77	Basques, Spanish ...	164	79
French	165	84	Spaniards	164	79
Basques	166	83	Sundanese	159	86
Fulahs	174	75	Tamils	165	76
Gypsies	165	80	„ S. India ...	167	77
Hindus	163	73	Tenggerese	160	80
„ Behar	163	76	Tipperahs	161	80
Iroquois	173	79	Toucouleurs	172	74
Italians	164	83	Trao Mois	158	77
Javanese	162	85	Uru-Kurubas	164	76
Jakuns	153	81	Ute Indians	166	79
Kharvars	162	76	Veddahs	155	75
Kothas	163	74	Wolofs	173	75
Kols	165	72	Zuñis	162	83
Kurmis	161	76	Zandehs, W.	172	78

TABLE III.

Cranial Indices.

Race or Group	Cephalic Index	Upper Face Index	Race or Group	Cephalic Index	Upper Face Index
Negroes	74·3	51·3	Hottentots	72·4	52·1
Australians	71·0	79·3	Tasmanians	75·8	46·3
Javanese	81·5	49·7	Torres Straits ...	68·6	52·6
Marquisans	75·8	51·6	Viti-Levu	66·8	48·2
Auvergnats	86·3	49·8	Maoris	74·0	49·4
Esthonians	78·6	53·1	New-Caledonians ...	71·6	48·6
Dawson Straits ...	74·2	49·5	Eskimos	71·8	51·7
Württembergians ...	82·6	53·0	,, Greenland ...	71·4	56·1
Bavarians	82·7	52·6	Veddahs	69·9	49·9
Chinese	79·0	54·0	Andamanese	81·5	48·7
Ainos	76·5	50·0	Malays	80·2	50·2
Kalmuks	81·6	53·2	Polynesians	77·7	50·2
Minusinsk	73·2	53·1	Alfurous	80·7	50·9
,, ...	74·6	52·5	Micronesians	73·4	51·1
French	79·5	51·9	Papuans	72·4	49·5
Bashkirs	81·0	52·1	Czechs	83·2	51·4
Duke of York Islanders	70·4	48·7	Telenguts	86·1	51·0
Kanakas	78·8	49·8	Soiots	86·3	53·2
Celebes	82·1	51·8	Samoyeds	82·4	53·2
Naqada	72·9	53·8	Tarbagatai-Torguts ...	83·3	52·2
Thebans	75·1	53·5	Abyssinians	75·7	52·3
Copts	77·7	54·5	Senftenberg Crania ...	84·2	49·5
Etruscans	78·1	54·1	Rumanians	80·6	52·3
Kalmuks	81·2	55·2			

When a race is repeated, two series have been used.

TABLE IV. *Cranial Measurements of Various Races.*

Race or Group	Authority	Cephalic Index	Length	Breadth	Height	Upper Face Height	Length of Basis	Minimum Frontal Breadth	Upper Face Height	Bizygomatic Breadth
Negroes	Different Authorities ...	72·3	183·3	132·3	133·9	101·9	117·9	95·7	67·8	130·3
Australians	Smidt, Sandfroi, Flower, etc.	71·0	185·7	132·2	134·2	102·1	118·7	96·3	65·7	133·2
Javanese	Davis, Flower, etc.	81·5	174·2	141·9	145·4	99·3	121·2	93·9	67·2	124·3
Dawson Straits	Sergi	—	177·3	131·6	134·4	—	—	91·7	63·7	128·6
Württembergians... ...	„	82·6	179·8	147·4	131·3	98·6	—	98·2	71·1	134·1
Torguts	Iwanowsky	81·6	179·0	146·0	129·0	96·0	—	96·0	73·0	138·0
Minousinsk	Goroshenko	73·2	186·2	138·3	135·7	101·8	—	102·1	74·8	136·8
Duke of York Islanders ...	Godefroi	—	180·1	132·0	137·0	100·1	118·1	92·0	68·5	133·4
Kanakas	Davis	—	181·9	142·8	141·8	103·8	124·9	94·2	67·7	134·5
Polynesians	Flower, Davis, Godefroi, etc.	—	183·6	140·7	138·0	104·5	125·1	94·5	68·3	136·5
Viti-Levu	Flower	66·8	190·9	127·6	140·1	102·2	115·1	94·3	64·8	131·2
Naqada	Fawcett	72·9	184·6	134·8	135·2	99·3	—	91·1	67·6	125·6
Thebans	„	75·1	181·9	136·6	136·0	100·6	—	93·8	68·8	128·3
Copts	„	77·2	177·0	136·8	137·2	100·9	—	94·1	68·7	125·8
Bavarians	Ranke	83·2	180·6	150·5	133·8	100·3	—	103·7	70·8	135·0
French	Koernef	—	180·4	144·1	131·0	99·7	—	96·2	68·2	131·3
Bashkirs	Nicolsky	81·0	174·4	140·8	129·3	97·0	—	95·4	68·8	132·0
Ainos	Koganei	76·5	183·8	141·2	139·5	105·4	—	96·2	69·8	137·3
Etruscans	Schmidt	79·8	182·8	142·8	139·9	105·0	122·3	—	70·9	131·2
Berbers	Crania Ethnica	73·9	184	136	138	101	—	96	—	129·0
Tahitians	„	75·1	185	139	143	106	—	94	—	133
Sungarians	„	78·2	179	140	143	102	—	92	—	131
Loyalty Islanders... ...	„	69·8	189	132	139	104	—	96	—	136
Kioways	„	75·5	184	139	141	105	—	94	—	134
Andamanese	Flower	81·5	167·3	136·5	129·3	93·9	113·4	93·0	60·0	123·6
Veddahs	„	69·9	178·4	124·8	133·6	98·7	110·3	90·6	60·1	121·9
New Caledonians... ...	Crania Ethnica	69·7	188	131	170	104	—	96	—	135
„ „ Kanala...	„	71·3	185	132	139	104	—	97	—	137
Arabs	„	73·0	185	135	135	103	—	99	—	126
Tarbagataï	Iwanowsky	83·3	177	148	122	90	—	94	72	135
Chinese	Koganeï	78·3	180·1	140·5	140·5	100·4	—	91·9	75·2	135·6
Eskimos	Crania Ethnica	71·8	188	135	139	106	—	95	—	135
Auvergnats	Broca	—	178·1	154·0	131·0	97·2	—	—	—	—
Basques	„	—	185·8	142·3	131·6	99·2	—	—	—	—
Hottentots	„	71·4	182·8	130·3	132·6	100·2	—	—	66·5	128·1
Papuans	Godefroi, etc.	—	179·6	121·7	134·3	99·9	120·9	93·7	65·1	133·8
Micronesians	Different Sources	—	180·0	133·8	136·6	105·8	123·6	—	67·8	128·2
Abyssinians	Koernef	—	181	137	136	102	—	95	68	130
Nordlander (Netherlands)	„	—	175	144·5	131	97	—	97	65	131
Tasmanians	Flower, etc.	—	185·3	139·3	133·0	100·5	121·6	95·1	60·6	131·3
Torres Straits	Thomas	68·3	190·6	130·0	135·4	103·4	—	98·3	71·5	135·2
Marquesans	R. College Skulls	—	185·9	141·1	137·9	104·5	125·3	95·0	70·0	136·8
Maoris	„	74·0	189·8	137·1	139·6	104·3	126·7	96·6	68·1	137·8

Upper Face Index.

APPENDIX II. CORRELATION TABLES.

TABLE I. *Russian Races*, cf. Table II. p. 300.

Head Index.

Stature.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	Totals
154	—	—	—	—	—	—	—	1	1	—	—	—	—	—	2
155	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
156	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
157	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
158	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
159	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	1	—	—	3	1	—	—	—	—	—	—	5
162	—	—	—	—	—	1	4	1	2	—	—	—	—	—	8
163	—	—	1	—	—	—	1	1	1	—	3	1	—	—	8
164	—	—	1	1	—	2	2	2	1	1	1	2	1	—	14
165	—	—	1	—	1	2	2	3	—	1	—	2	—	1	13
166	1	—	—	—	1	1	2	1	1	1	—	—	—	—	8
167	—	—	1	—	2	1	1	1	1	—	1	1	—	—	9
168	1	—	2	1	—	—	2	—	1	—	—	2	—	—	9
169	—	—	1	—	—	1	—	—	—	1	—	—	—	—	3
170	1	—	—	1	1	—	1	—	1	—	—	—	—	—	5
171	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
172	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
173	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	3	1	7	5	6	9	20	11	10	4	6	8	1	1	92

(For Table II. see p. 300.)

TABLE III.

Head Index.

Face Index.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	Totals
72	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
73	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
74	—	—	1	—	—	1	1	—	—	—	—	—	—	—	3
75	1	—	1	—	2	—	—	3	—	—	—	—	—	—	7
76	—	—	1	—	1	—	1	—	—	—	—	2	—	—	5
77	—	—	—	—	1	2	3	1	1	—	—	3	—	—	11
78	—	1	—	—	—	—	1	—	—	—	1	1	—	—	4
79	—	—	—	—	—	1	4	1	—	—	1	1	—	—	8
80	—	—	—	1	—	—	1	—	1	1	2	1	—	1	8
81	—	—	—	—	—	1	1	—	—	—	—	—	—	—	2
82	—	—	—	—	—	1	—	—	—	—	—	1	—	—	2
83	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
84	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
85	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	1	1	3	1	4	6	13	5	4	2	4	9	—	1	54

TABLE IV.

Cranial Cephalic Index.

Upper Face Index.	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	Totals
46	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
47	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
48	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
49	—	—	—	—	2	2	—	1	1	—	—	—	—	—	1	—	—	1	—	—	8
50	—	—	—	1	—	—	—	—	—	1	—	1	1	1	1	—	—	—	—	1	7
51	—	—	—	—	—	—	1	1	—	—	—	—	—	—	1	—	1	—	—	1	5
52	—	—	1	—	—	2	—	—	1	2	—	—	1	—	2	1	1	—	—	—	11
53	—	—	—	—	—	—	1	—	1	—	—	—	1	—	—	2	2	—	—	1	8
54	—	—	—	—	—	—	1	—	—	—	—	2	1	—	—	—	—	—	—	—	4
55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
56	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	1	1	3	4	3	2	3	4	—	3	4	1	6	3	4	1	—	3	47

TABLE II. *Non-Russian Races.*

Stature.

Head Index.	146	147	148	—	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	Totals
72	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	—	—	—	—	3
73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	2
74	—	—	—	—	—	—	—	—	—	—	—	—	1	—	3	1	—	—	2	1	—	—	—	1	—	—	9
75	—	—	—	—	—	—	—	2	—	—	1	—	2	—	—	—	—	—	—	—	—	1	—	—	1	1	8
76	—	—	—	—	—	—	—	—	—	1	—	—	2	1	1	1	1	1	—	—	—	—	—	—	—	—	8
77	—	—	—	—	—	—	—	1	—	1	—	1	1	4	1	—	—	—	1	—	1	—	—	—	—	—	11
78	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	2
79	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	1	—	1	—	—	—	—	—	—	1	—	6
80	—	—	—	—	—	—	—	—	1	—	—	1	—	—	1	—	1	1	—	—	—	—	1	—	3	—	9
81	—	—	1	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
82	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	2
83	—	—	—	—	—	—	—	—	1	1	—	—	—	1	1	1	1	1	—	—	—	—	—	1	—	—	8
84	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	1	—	—	—	—	—	—	—	—	4
85	1	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	—	—	—	—	1	—	5
86	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2
87	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2
Totals	1	—	1	—	2	—	—	3	2	3	3	4	6	8	11	4	7	7	3	2	1	2	3	3	7	1	84

(For Table IV. see p. 299.)

TABLE V.

Nasal Index.

Head Index.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	81	Totals
76	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
77	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
78	—	—	1	—	—	—	—	—	—	—	—	2	1	—	—	—	1	1	—	—	—	—	—	—	—	—	—	6
79	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	2
80	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	1	3
81	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	1	—	—	—	1	—	—	—	—	4
82	—	1	—	—	—	—	2	—	—	1	—	1	1	—	—	—	—	2	—	1	1	—	2	—	—	—	—	12
83	—	1	—	—	—	—	1	—	1	—	—	—	—	1	—	—	1	—	1	1	1	—	1	—	—	1	—	10
84	—	—	—	1	1	—	1	—	—	1	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	6
85	—	—	—	—	—	1	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
86	—	—	—	—	—	—	1	1	—	1	—	1	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	6
87	—	—	—	—	—	—	—	1	—	—	—	—	2	1	2	1	—	—	—	—	—	—	—	—	—	1	—	8
88	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
89	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	1	3	1	1	1	1	5	2	1	3	—	5	6	3	4	2	3	4	4	2	2	3	4	1	—	2	1	65

Stature.

Stature.

TABLE VI.

Length of the Head.

Stature.	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	Totals
154	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
155	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
156	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
157	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
158	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
159	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	—	—	—	—	2	—	1	—	—	1	—	—	—	—	—	—	—	—	4
162	—	—	—	1	—	—	2	—	1	2	—	—	—	—	1	—	—	—	—	—	—	7
163	1	—	—	1	—	—	1	—	—	—	2	1	—	1	—	1	—	—	—	—	—	8
164	—	—	—	—	1	—	2	2	—	2	1	—	2	1	—	—	—	1	—	—	—	12
165	—	—	1	—	1	2	1	2	1	2	—	1	1	—	—	—	—	—	—	—	1	13
166	—	—	1	—	—	—	1	—	—	1	1	2	—	—	1	—	—	—	—	—	—	7
167	1	—	—	—	1	1	—	1	—	1	1	—	—	1	1	—	—	—	—	—	—	8
168	—	—	—	—	—	1	1	—	—	3	—	—	2	—	—	1	1	—	—	—	—	9
169	—	—	—	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	—	—	3
170	—	—	1	—	—	—	—	—	1	—	—	—	—	1	—	—	1	—	—	—	—	4
171	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
172	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
173	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	2	—	3	2	3	4	10	9	4	12	5	4	7	7	3	3	2	2	—	—	1	83

TABLE VII.

Head Breadth.

Stature.	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	Totals
154	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
155	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
156	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
157	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
158	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
159	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	1	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	—	—	4
162	—	—	—	—	—	1	1	2	—	—	—	—	—	1	2	—	—	—	—	—	—	7
163	—	—	—	—	—	1	—	—	—	1	—	1	1	—	—	1	1	1	—	1	—	8
164	—	—	1	—	1	—	—	1	—	1	2	1	—	2	1	—	—	1	1	—	—	12
165	—	—	—	—	—	1	—	—	2	2	1	2	2	—	—	—	—	—	1	1	1	13
166	1	—	—	—	—	—	2	—	—	1	1	—	1	1	—	—	—	—	—	—	—	7
167	—	—	—	1	—	—	—	—	—	2	1	—	1	1	1	—	—	—	—	1	—	8
168	1	1	—	—	—	—	1	—	—	1	1	—	—	1	—	2	1	—	—	—	—	9
169	—	—	—	—	1	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	3
170	—	—	—	1	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	4
171	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
172	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
173	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	1	2	2	2	3	4	5	4	12	8	6	7	7	5	3	2	2	2	3	1	83

TABLE VIII.

Head Height.

Stature.

	115	*116*	*117*	*118*	—	*121*	*122*	*123*	*124*	*125*	*126*	*127*	*128*	*129*	*130*	*131*	*132*	*133*	*134*	*135*	*136*	*137*	*138*	*139*	*140*	*141*	—	*144*	*145*	*146*	Totals
155	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
156	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
157	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
158	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
159	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
162	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	3
163	—	—	—	—	—	—	1	—	—	1	—	1	—	—	1	—	—	—	—	—	1	—	—	1	—	1	—	—	—	—	7
164	—	—	—	—	—	1	—	—	—	—	—	1	1	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	7
165	—	—	—	—	—	1	1	—	—	—	—	—	—	2	1	2	1	—	—	—	1	1	—	—	—	—	—	—	—	1	11
166	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	3
167	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	2	—	—	1	—	—	—	—	—	1	—	—	7
168	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	—	1	1	—	—	—	—	—	5
169	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
170	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	1	1	—	2	3	—	1	2	1	2	1	3	4	4	5	3	3	2	5	3	—	2	1	1	—	1	—	1	53

TABLE IX.

Bizygomatic Breadth.

Stature.	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	—	151	152	—	159	Totals
156	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
157	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
158	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
159	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2
162	—	—	—	1	1	1	1	—	—	—	—	—	—	—	1	—	—	—	—	—	5
163	—	—	—	—	—	1	—	—	—	—	1	—	—	1	—	—	2	1	—	1	7
164	1	—	—	1	1	1	1	1	—	—	2	—	—	1	—	—	1	—	—	—	10
165	—	—	—	1	—	2	2	2	—	2	—	1	—	—	1	—	—	—	—	—	11
166	1	—	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
167	—	—	—	—	—	—	1	1	—	1	1	—	—	—	—	—	—	—	—	—	4
168	—	—	—	—	—	—	3	—	—	1	—	1	1	—	—	—	—	—	—	—	6
169	—	—	—	—	1	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	3
170	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
171	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
172	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	—	1	5	3	7	8	5	1	5	5	4	2	2	2	—	4	1	—	1	58

TABLE X.

Face Index.

Stature.	72	73	74	75	76	77	78	79	80	81	82	—	85	Totals
156	—	—	—	—	—	—	—	1	—	—	—	—	—	1
157	—	—	—	—	—	—	1	—	—	—	—	—	—	1
158	—	—	—	—	—	—	—	—	—	—	—	—	—	—
159	—	—	—	—	—	—	—	—	1	—	—	—	—	1
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	1	—	—	1	—	—	—	—	—	—	—	2
162	—	—	—	—	1	—	—	2	—	1	—	—	—	4
163	—	—	—	—	1	—	—	—	1	1	—	—	1	4
164	1	1	—	1	—	—	—	3	2	—	—	—	—	8
165	—	—	—	2	1	3	—	1	2	—	—	—	—	9
166	—	—	—	3	—	1	—	—	—	—	—	—	—	4
167	—	—	—	—	1	1	2	—	—	—	—	—	—	4
168	—	—	1	—	1	1	—	1	1	—	1	—	—	6
169	—	1	—	1	—	—	—	—	—	1	—	—	—	3
170	—	—	—	—	—	—	—	—	1	—	—	—	—	1
171	—	—	—	—	—	—	1	—	—	—	—	—	—	1
172	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	1	2	3	7	5	7	4	8	8	3	1	—	1	50

TABLE XI.

Skull Length.

Skull Breadth.

	167	—	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	Totals
122	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
124	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
126	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
129	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
130	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	2
131	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
132	—	—	—	—	—	1	—	—	1	—	—	1	—	1	1	—	—	1	—	—	6
133	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
134	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
135	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	3
136	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
137	—	—	—	—	—	1	—	—	—	1	1	—	—	—	—	—	—	—	1	—	4
138	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
139	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	—	—	3
140	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	2
141	—	—	1	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	—	—	4
142	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2
143	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
144	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
146	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
147	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
148	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
150	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
154	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	2	1	—	3	2	2	6	2	2	3	3	5	5	—	2	1	1	2	43

TABLE XII.

Skull Length.

Skull Height.

	167	—	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	Totals
122	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
129	1	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
130	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
131	—	—	—	1	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	4
132	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
133	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	2
134	—	—	—	—	—	1	1	—	1	1	—	1	—	—	1	—	—	—	—	—	6
135	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	1	3
136	—	—	—	—	—	—	—	—	—	1	1	—	—	—	1	—	—	—	—	—	3
137	—	—	—	—	—	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	3
138	—	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	3
139	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	1	1	—	—	5
140	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	1	1	4
141	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
142	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
143	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	2
144	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
145	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	2	1	—	3	2	2	6	2	2	3	3	5	5	—	2	1	1	2	43

TABLE XIII.

TABLE XIII.

Skull Breadth.

Skull Height. Biometrika IV

	122	—	125	—	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	—	154	Totals
122	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	3
129	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—
130	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	4
131	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	1
132	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
133	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	6
134	1	—	1	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
135	—	—	—	—	—	—	1	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
136	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
137	—	—	—	—	—	—	—	—	1	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
138	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	5
139	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	4
140	—	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
141	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
143	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	2
144	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	—	1	—	1	—	2	1	6	—	1	3	2	4	1	3	2	4	2	2	2	—	1	1	1	—	1	—	1	43

(For Table XIV. see p. 306.)

TABLE XVI.

Skull Breadth.

Upper Face Height.

	122	—	125	—	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	Totals
60	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
61	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
65	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	3
66	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
67	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
68	—	—	—	—	—	—	—	—	2	—	1	1	—	2	—	—	—	1	—	1	1	—	—	—	—	—	—	9
69	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	1	—	—	—	—	—	—	—	—	—	3
70	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	2
71	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	1	4
72	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	1
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	2
Totals	1	—	1	—	1	—	2	—	4	—	1	1	1	4	1	1	1	4	1	2	2	—	1	1	1	1	—	32

TABLE XIV.

Upper Face Height.

Bizygomatic Breadth.	*60*	*61*	—	*64*	*65*	*66*	*67*	*68*	*69*	*70*	*71*	*72*	*73*	*74*	*75*	Totals
122	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
124	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	2
125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
126	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	3
129	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
130	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
131	—	1	—	—	2	—	—	1	—	—	1	—	—	—	—	5
132	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
133	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
134	—	—	—	—	1	—	—	1	—	—	1	—	—	—	—	3
135	—	—	—	—	—	—	—	—	—	—	2	1	—	—	—	3
136	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	2
137	—	—	—	—	—	—	—	—	—	2	—	—	—	—	1	3
138	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	2
Totals	2	1	—	1	3	2	1	9	3	2	4	1	1	—	2	32

TABLE XV.

Skull Length.

Upper Face Height.	*167*	—	*174*	*175*	*176*	*177*	*178*	*179*	*180*	*181*	*182*	*183*	*184*	*185*	*186*	—	*190*	*191*	Totals
60	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
61	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
65	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	1	3
66	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2
67	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
68	—	—	—	—	—	—	—	—	3	1	1	1	1	1	—	—	1	—	9
69	—	—	1	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	3
70	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	2
71	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	1	4
72	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
73	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	2
Totals	1	—	2	1	—	3	1	1	6	2	2	3	1	2	4	—	1	2	32

(For Table XVI. see p. 305.)

Bizygomatic Breadth.

TABLE XVII.

Skull Height.

Upper Face Height.

	122	—	*129*	*130*	*131*	*132*	*133*	*134*	*135*	*136*	*137*	*138*	*139*	*140*	*141*	*142*	—	*145*	Totals
60	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
61	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
65	—	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	—	—	3
66	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	2
67	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
68	—	—	—	—	1	—	—	1	1	1	2	1	—	1	—	1	—	—	9
69	—	—	1	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	3
70	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	2
71	—	—	—	—	1	—	—	1	1	—	—	—	1	—	—	—	—	—	4
72	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
73	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
Totals	1	—	3	—	3	—	2	6	2	3	3	2	2	3	—	1	—	1	32

TABLE XVIII.

Skull Length.

Bizygomatic Breadth.

	167	—	*174*	*175*	*176*	*177*	*178*	*179*	*180*	*181*	*182*	*183*	*184*	*185*	*186*	*187*	*188*	*189*	*190*	*191*	Totals
122	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
124	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
126	—	—	—	—	—	1	—	—	—	—	—	—	—	2	—	—	—	—	—	—	3
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	3
129	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
130	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	2
131	—	—	—	1	—	—	—	1	1	—	—	1	—	1	—	—	—	—	—	1	6
132	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
133	—	—	—	—	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	3
134	—	—	—	—	—	—	—	—	2	—	1	—	1	—	—	—	—	—	—	—	4
135	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	2	—	—	1	5
136	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	1	—	—	3
137	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3	—	—	—	—	—	4
138	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	2
Totals	1	—	2	1	–	3	1	2	6	2	2	3	3	5	4	—	2	1	1	2	41

TABLE XIX.

Skull Breadth.

Bizygomatic Breadth.	122	—	125	—	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	Totals
122	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	2
124	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	1	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
129	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
130	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
131	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	1	—	—	1	2	—	—	—	—	—	—	6
132	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
133	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	3
134	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	1	—	—	—	4
135	—	—	—	—	—	—	1	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	5
136	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	3
137	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	2	—	—	—	—	1	—	—	—	—	4
138	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	1	—	1	—	1	—	2	1	6	—	1	3	2	4	1	3	2	4	1	2	2	—	1	1	1	—	1	41

TABLE XX.

Skull Height.

Bizygomatic Breadth.	122	—	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	Totals
122	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
124	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	—	—	3
126	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
128	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	—	—	2
129	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	2
130	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	6
131	—	—	—	—	2	—	1	—	—	—	—	—	—	1	1	—	—	1	—	1
132	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
133	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	1	—	1	—	4
134	—	—	—	—	1	—	—	1	—	—	—	—	—	1	1	1	1	—	—	5
135	1	—	—	—	—	—	—	1	1	—	—	—	1	1	1	—	—	—	—	3
136	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	—	4
137	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	2
138	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	—	3	—	3	—	2	6	3	3	3	3	5	4	1	1	2	—	1	41

TABLE XXI.

Minimum Frontal Breadth.

Skull Length.	91	92	93	94	95	96	97	98	99	—	102	103	104	Totals
167	—	—	1	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	1	1	—	—	—	—	—	—	—	—	2
175	—	—	—	—	—	—	1	—	—	—	—	—	—	1
176	—	—	—	—	—	—	—	—	—	—	—	—	—	—
177	—	1	—	2	—	—	—	—	—	—	—	—	—	3
178	1	—	—	—	—	—	—	—	—	—	—	—	—	1
179	—	1	—	—	—	1	—	—	—	—	—	—	—	2
180	—	2	—	1	—	1	—	1	—	—	—	—	—	5
181	—	—	—	—	1	—	—	—	—	—	—	—	1	2
182	—	—	—	2	—	—	—	—	—	—	—	—	—	2
183	—	—	—	—	—	1	—	—	—	—	—	—	—	1
184	—	—	—	2	—	1	—	—	—	—	—	—	—	3
185	1	—	—	1	1	—	1	—	1	—	—	—	—	5
186	—	—	—	—	1	2	—	—	—	—	1	—	—	4
187	—	—	—	—	—	—	—	—	—	—	—	—	—	—
188	—	—	—	—	1	1	—	—	—	—	—	—	—	2
189	—	—	—	—	—	1	—	—	—	—	—	—	—	1
190	—	—	—	—	—	—	1	—	—	—	—	—	—	1
191	—	—	—	1	—	—	—	1	—	—	—	—	—	2
Totals	2	4	1	10	5	8	3	2	1	—	1	—	1	38

TABLE XXII.

Minimum Frontal Breadth.

Skull Breadth.	91	92	93	94	95	96	97	98	99	—	102	103	104	Totals
122	—	—	—	1	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
125	1	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	1	—	—	—	—	—	—	—	—	—	1
129	—	—	—	—	—	—	—	—	—	—	—	—	—	—
130	—	—	—	—	—	—	—	1	—	—	—	—	—	1
131	—	—	—	—	—	1	—	—	—	—	—	—	—	1
132	—	2	—	—	—	3	1	—	—	—	—	—	—	6
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
135	1	—	—	—	1	—	—	—	1	—	—	—	—	3
136	—	—	1	—	—	1	—	—	—	—	—	—	—	2
137	—	—	—	2	1	—	1	—	—	—	—	—	—	4
138	—	—	—	—	—	—	—	—	—	—	1	—	—	1
139	—	—	—	2	1	—	—	—	—	—	—	—	—	3
140	—	2	—	—	—	—	—	—	—	—	—	—	—	2
141	—	—	—	1	2	1	—	—	—	—	—	—	—	4
142	—	—	—	1	—	—	—	—	—	—	—	—	—	1
143	—	—	—	1	—	—	—	—	—	—	—	—	—	1
144	—	—	—	—	—	1	1	—	—	—	—	—	—	2
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—
146	—	—	—	—	—	1	—	—	—	—	—	—	—	1
147	—	—	—	—	—	—	—	1	—	—	—	—	—	1
148	—	—	—	1	—	—	—	—	—	—	—	—	—	1
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—
150	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	4	1	10	5	8	3	2	1	—	1	—	1	38

TABLE XXIII.

Minimum Frontal Breadth.

Skull Height.	*91*	*92*	*93*	*94*	*95*	*96*	*97*	*98*	*99*	—	*102*	*103*	*104*	Totals
122	—	—	—	1	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
129	—	—	1	—	1	1	—	—	—	—	—	—	—	3
130	—	—	—	—	—	—	—	—	—	—	—	—	—	—
131	—	—	—	—	—	1	1	1	—	—	—	—	—	3
132	—	—	—	—	—	—	—	—	—	—	—	—	—	—
133	—	—	—	—	1	—	—	—	—	—	—	—	—	1
134	1	1	—	1	—	2	—	—	—	—	—	—	1	6
135	1	—	—	—	—	—	—	1	1	—	—	—	—	3
136	—	—	—	1	1	—	—	—	—	—	1	—	—	3
137	—	1	—	1	—	—	—	—	—	—	—	—	—	2
138	—	—	—	1	1	1	—	—	—	—	—	—	—	3
139	—	—	—	—	1	2	1	—	—	—	—	—	—	4
140	—	1	—	1	—	1	1	—	—	—	—	—	—	4
141	—	—	—	1	—	—	—	—	—	—	—	—	—	1
142	—	—	—	1	—	—	—	—	—	—	—	—	—	1
143	—	1	—	1	—	—	—	—	—	—	—	—	—	2
144	—	—	—	—	—	—	—	—	—	—	—	—	—	—
145	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	2	4	1	10	5	8	3	2	1	—	1	—	1	38

TABLE XXIV.

Skull Basis.

Naso-Alveolar Height.	*90*	—	*94*	*95*	*96*	*97*	*98*	*99*	*100*	*101*	*102*	*103*	*104*	*105*	*106*	Totals
60	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	2
61	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65	—	—	—	—	—	1	—	—	1	—	1	—	—	—	—	3
66	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
67	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
68	—	—	—	—	—	—	—	1	2	—	1	—	3	—	1	8
69	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	3
70	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	3
71	—	—	—	—	—	—	—	1	1	—	—	1	—	1	—	4
72	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
73	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
Totals	1	—	1	—	1	2	—	4	7	2	5	1	4	2	1	31

TABLE XXV.

Skull Basis.

Skull Breadth.

	90	—	94	95	96	97	98	99	100	101	102	103	104	105	106	Totals
122	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
125	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
129	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
130	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2
131	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
132	—	—	—	—	—	—	—	—	1	—	2	—	2	—	—	5
133	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
134	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
135	—	—	—	—	—	—	—	1	—	—	—	1	—	—	1	3
136	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	2
137	—	—	—	—	—	—	—	—	—	2	1	—	1	—	—	4
138	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
139	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	3
140	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
141	—	—	—	—	—	1	—	—	—	—	—	—	2	1	—	4
142	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
143	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	2
144	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	2
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
146	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
147	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
148	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
150	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
154	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	1	—	1	—	1	3	—	5	6	4	6	2	7	3	3	42

TABLE XXVI.

Skull Basis.

Bizygomatic Breadth.

	90	—	94	95	96	97	98	99	100	101	102	103	104	105	106	Totals
122	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
123	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
124	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	2
125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
126	—	—	—	—	—	—	—	1	—	1	—	1	—	—	—	3
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	3
129	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
130	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	2
131	—	—	—	—	—	1	—	—	2	—	2	—	—	1	—	6
132	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
133	—	—	—	—	—	—	—	—	1	—	1	—	—	—	1	3
134	—	—	—	—	—	—	—	1	1	—	—	—	1	1	—	4
135	1	—	—	—	—	—	—	—	1	—	—	1	1	—	1	5
136	—	—	—	—	—	—	—	—	1	—	—	—	2	—	—	3
137	—	—	—	—	—	—	—	—	—	—	1	—	2	1	—	4
138	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	2
Totals	1	—	1	—	1	2	—	4	7	3	6	2	7	3	3	40

TABLE XXVII.

Skull Basis.

Skull Height.

	90	—	*94*	*95*	*96*	*97*	*98*	*99*	*100*	*101*	*102*	*103*	*104*	*105*	*106*	Totals
122	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
129	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	3
130	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
131	—	—	—	—	—	2	—	1	1	—	—	—	—	—	—	4
132	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
133	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
134	—	—	—	—	—	—	—	1	2	—	2	—	—	—	—	5
135	—	—	—	—	—	—	—	1	—	—	—	2	—	—	—	3
136	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	3
137	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	3
138	—	—	—	—	—	—	—	—	—	1	—	—	2	—	—	3
139	—	—	—	—	—	—	—	—	—	—	—	—	2	2	1	5
140	—	—	—	—	—	—	—	—	1	—	1	—	2	—	—	4
141	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
142	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
143	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	2
144	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
145	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	1	—	1	—	1	3	—	5	7	3	6	2	7	3	3	42

TABLE XXVIII.

Skull Basis.

Skull Length.

	90	—	*94*	*95*	*96*	*97*	*98*	*99*	*100*	*101*	*102*	*103*	*104*	*105*	*106*	Totals
167	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
174	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
175	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
176	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
177	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2
178	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
179	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	2
180	—	—	—	—	—	—	—	1	4	—	—	—	—	—	1	6
181	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
182	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	2
183	—	—	—	—	—	—	—	—	1	—	1	—	—	1	—	3
184	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	3
185	—	—	—	—	—	—	—	1	1	—	—	1	1	—	1	5
186	—	—	—	—	—	—	—	1	—	—	2	—	1	1	—	5
187	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
188	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2
189	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
190	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
191	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	2
Totals	1	—	1	—	1	3	—	5	7	3	6	2	7	3	3	42

STATISTICAL STUDIES IN IMMUNITY.

SMALLPOX AND VACCINATION.

By JOHN BROWNLEE, M.A., M.D., Glasgow.

In writing this paper on smallpox and vaccination I have no intention of entering into any controversy upon the subject. The fact that vaccination protects against smallpox is assumed, and consideration is alone given here to the relationship between vaccination, revaccination, and smallpox, in so far as the facts throw light upon the growth, decline, or establishment of immunity in persons who have passed through one or other of these infections. I would rather have chosen for this investigation some other disease, but as an inquiry of this kind is essentially statistical there was no other series of figures available. The disadvantage of choosing vaccination and smallpox arises from the fact that the former is to a considerable extent different from the latter, and though there seems good reason for believing that vaccination is a disease caused by the parasite of smallpox so modified as to have lost one stage in its life history, yet the fact that it protects chiefly against the second stage of smallpox causes the immunity relationship to be of a more complex character than is desirable.

Before proceeding, however, to consider the relationship of smallpox to vaccination one fact demands special preliminary notice: that is the change which has taken place in the age at which vaccinated persons are attacked by smallpox. All over the country from the earliest period of last century for which statistics are available there has been a steady increase in the mean age at which such persons have been attacked by smallpox.

This is quite distinct from the rise in the mean age at death, shown by the combined mortality statistics of the vaccinated and unvaccinated which is commonly and correctly looked upon as largely due to the protective influence of vaccination upon the younger members of the community. What I refer to is a process which applies specially to the vaccinated, and which does not seem to admit of any complete explanation from the point of view of alteration in the character of the population. In London, for instance, between the years 1836

and 1851, of 3,094 cases 2,719 were under the age of thirty, or more than seven vaccinated persons under thirty were affected with smallpox for each individual above that age. In Dr Gayton's statistics relating to the period 1870 to 1883 the corresponding proportion is 5½ to 1. In Dr Sweeting's referring to 1880 to 1885 the proportion is 3 to 1, while in the epidemics of the last ten years two

TABLE I.

Age of Smallpox Attack in Case of Vaccination.

Age Periods	London, 1836–51		London, 1870–72		London, 1880–85		London, 1892–95		London, 1902	
	Cases	Deaths	Cases	Deaths	Cases	Deaths	Cases	Deaths	Cases	Deaths
0—5	7	2	195	38	26	—	15	—	16	—
5—10	56	7	786	60	108	2	101	—	98	2
10—15	206	10	4504	238	256	8	188	2	265	3
15—20	866	49			405	10	533	5	654	17
20—25	1058	93	3761	410	632	59	639	16	1049	45
25—30	526	55					454	16	1017	70
30—35	210	21	1297	257	455	68	370	14	827	91
35—40	102	20					234	14	616	111
40—50	61	10	435	110			293	8	726	154
50—60	2	1	134	30			89	8	254	52
60—70	—	—	62	12			24	7	105	27
70—							10	1	36	6
0—30	2719	216	9246	756	1427	79	1950	39	3099	137
30—	375	52	1928	389	455	68	1020	52	2564	441
Mean Age	22·6	24·5	21·9	26·4			26·4	33·4	28·6	38·2

persons under thirty years only are affected for each one over that age. In every town where there are statistics for a series of outbreaks of smallpox during the last thirty years exactly the same fact is observed. To mention Glasgow; in 1872 this proportion was 6 to 1, in 1892 to 1895 2½ to 1, and during the epidemic of 1901 the ratio reached unity. Now there can be no doubt that a part of this is due to the gradual disappearance of adult persons protected by previous attacks of smallpox. In addition during the years immediately succeeding Jenner's discovery vaccination was performed promiscuously on young and old alike, and persons vaccinated in adult years must have constituted during the first sixty years of the century a very considerable proportion of the total vaccinated persons at higher ages, as it can easily be seen that any person in 1836 whose age was greater than thirty-seven must have been vaccinated at the earliest at an age corresponding to the difference of the actual age and of thirty-seven years. By 1870, however, both these influences must have to a considerable extent disappeared, and yet we see that the change referred to has been going on just as markedly since that date. Against these two factors must be set one which undoubtedly acts in the opposite direction, and that is the gradual better enforcement of vaccination during infancy, which, by continually providing a larger number of vaccinated persons at lower ages, ought if no other change were taking place to supply a greater number of cases at these ages. The influence of this should at least be sufficient, at any rate since 1870, to neutralise that due

to the factors before mentioned. Unfortunately there is a complete absence of statistics referring to this matter, so that an examination of the relative influences of these different factors is impossible. The effect of revaccination on the age incidence of persons suffering from smallpox can be more readily estimated. When performed in the absence of an epidemic, or when slight outbreaks only of this disease are present, there is not a sufficient amount done to influence statistics much one way or another. In Sheffield, for instance, when the great epidemic of 1887 occurred, about five to seven per cent. of the population at each age period above fifteen years were found to be revaccinated. When, however, as in this epidemic in Sheffield and in the late epidemics in Glasgow revaccination is performed on a large scale, it is found that a much larger proportion of persons between ten and twenty years of age avail themselves of this means of protection than at higher ages. The effects of this on later epidemics are obviously, (1), if the second epidemic occurs within a few years of the first to raise the mean age; and (2), if the later epidemic occur about fifteen years thereafter to lower the mean age of attack. Revaccination, however, as a rule only begins to have an important modifying effect on the epidemic concerned when it is naturally approaching its end.

The change in the age incidence of smallpox just discussed is one which not only applies to the vaccinated as a class, but equally to the different groups comprehended in this class. Persons suffering from smallpox are for the purpose of classification commonly divided into groups as they present on their arms one, two, three, and four or more vaccinal cicatrices. In each of these groups the mean age has steadily risen. Now with regard to the group of those who present one cicatrix this is to be expected. Since 1870 persons vaccinated in this manner have been becoming progressively fewer and fewer, and consequently in London the bulk of those presenting one vaccinal cicatrix must consist of persons at the higher ages. Exactly the same change in the mean age, however, is found to be among the members of the group presenting one cicatrix in Glasgow, although no such change in the method of vaccination has taken place there. With regard to the group, however, of persons with four or more scars, year by year since 1870 a larger and larger number of such have come into existence at lower ages and consequently it would be expected that the mean age among this group would be at any rate stationary if not lower. This is not found to be the case. The annexed table exhibits the change in the mean age which has taken place in London since 1870. The rise noted in the mean age of persons with four marks

TABLE II.

Mean Age of Cases and Deaths in the Epidemics in London, 1870—1902.

	One Scar		Two Scars		Three Scars		Four Scars	
	Cases	Deaths	Cases	Deaths	Cases	Deaths	Cases	Deaths
1870–83	21·8	26·2	21·0	25·1	18·1	24·0	17·0	24·8
1892–95	26·8	35·3	29·5	43·9	26·5	31·7	23·2	35·7
1902	34·2	39·6	34·0	41·3	31·0	37·7	27·2	34·5

in view of the fact just mentioned seems to establish that this phenomenon is not due to alteration in the character of the population, but presents a biological as distinct from a statistical change. The explanations which can be offered from the biological point of view are three. Firstly, there may be some secular change taking place in the susceptibility of the population towards smallpox itself; secondly, there may be a like change taking place in the smallpox organism; and thirdly, it is possible that the passage of the organism of vaccinia through several thousand generations in the human subject has gradually increased the protective value of lymph; here again, however, there may be secular change. This increase in protective value, while it cannot be held proved, is in striking contradiction to what has been hitherto considered as probable. Plants propagated by cuttings, by tubers, and such like means, are well known to deteriorate in quality, and frequent recourse must be had to propagation by seeds if the strength of the species is to be maintained. To this rule, however, there are certain exceptions. Though it cannot be distinctly averred, however, that the vaccinal lymph has increased in potency, it can with all certainty be maintained that there is no evidence in favour of its deterioration. The evidence on which the argument regarding the deterioration of lymph was based cannot be regarded as complete. The chief points were, firstly, that as the century went on it became increasingly more possible to revaccinate successfully; and secondly, that in making attenuated forms of sheep-pox corresponding, as was thought, to vaccination, it was found that after a few generations these could not be relied on to give protection. In view of our fuller knowledge of the probable life history of the smallpox organism and its relations to vaccinia this latter argument ceases to present a reasonable analogy, while the former is more explicable on the view of the increasing potency of the lymph than on the other hypothesis.

The change just described is one which affects vaccinated persons as a whole and bears no special relationship to the question of the growth and decay of immunity. It is then to be noted that in all further discussions of smallpox statistics throughout the course of this paper the comparisons which are drawn, as far as possible concern persons in the same epidemic in the same place, because with different age constants at different places and at different times it can be seen what contradictory inferences could be made.

The first fact with regard to the immunity conferred by vaccination is one which seems unexpected. It is found that persons who are well protected have for all epidemics at all places a lower mean age than those who are badly protected. Exactly the same remarks apply to the groups which are based on the size or character of the cicatrix. Thus in each epidemic persons with four good marks have a lower mean age than those with four indifferent marks, a relationship which also applies to the group of persons presenting one, two, or three cicatrices. Taking the statistics of the large epidemics in Glasgow, of which more than half the cases consist of persons with one mark, it is found that the mean age in each of the three outbreaks falls steadily as the size of the mark

increases; that this is not due merely to the fact that larger marks are now put on than were formerly is shown moderately conclusively when it is considered further in detail. The degree of foveation of a cicatrix has been regarded as an indication of its protective value. Foveation, as a result of vaccination, no

TABLE III.

Mean Age of Persons with one Scar of different sizes, Glasgow 1901, 1902, *and* 1904.

Size of Scar	1901		1902*	1904
	Foveated	Not Foveated		
0—·25 sq. in.	32·7	33·7	38·3	37·3
·25—·5 „	31·6	32·1	34·8	33·3
·5—1 „	28·1	29·3	33·4	32·0
1— „	26·1	27·5	28·3	32·9

physician can foretell. The number, and, to a certain extent, the size of the cicatrices, is in the hands of the operator, but as to whether the scar will turn out to be foveated or not depends upon a number of factors beyond his control, and yet the mean age of those with unfoveated scars is for each group higher than that of those in which the scars are foveated. This relationship also applies uniformly as far as I have been able to observe, and in the large epidemics where there is a sufficient number of cases to allow of finer gradation the same relationship holds. For instance, taking those cases in the epidemic in London in 1902 in which there are four cicatrices of collective area greater than ½ square inch it is found that among those in which more than half of this area is foveated the mean age is lower than that of those in whom less than half is foveated, and that the mean age of the latter is in defect of that of those patients in whom there is no foveation, the cicatrix being glazed or puckered. I do not think that this can be explained as may be suggested on the ground that foveation is a feature of the scar which tends to disappear with age, for it must be noted that the relationship just described bears a constantly present inverse proportion to that observed to exist between the same subdivisions and the mortality: that is to say, many marks are a better protection against death than few, larger than smaller, and foveated than non-foveated, and this to a greater extent than the mere difference of age incidence would imply.

The susceptibility to smallpox of those with different numbers of marks is unfortunately a matter which cannot be determined directly, and consequently its full significance cannot be expressed in figures. It however can be investigated for London to a certain extent. At intervals of ten years since 1870 large outbreaks of smallpox have occurred in London. The number of persons affected at each age period with the number of their vaccinal scars is therefore known at four epochs separated by almost equal intervals of time, and consequently the way

* The incidence of mean age is here due to the fact that the revaccination in 1901 was performed more largely in the earlier than in the later ages.

in which persons are protected by different kinds of vaccination can be judged by a comparison of the proportions in each group at the same age period at the different epochs. Persons thus of the age five to ten years in 1872 will be fifteen to twenty in 1882, and will be thirty-five to forty-five in 1902. These proportions are recorded in the accompanying table. From this it may be seen that in general after forty years all vaccinated persons are apparently equally liable to smallpox. The proportions of persons attacked with smallpox who were from forty to sixty in 1872, sixty to eighty in 1892, and above seventy in 1902 with one, two, three, and four scars respectively, are sensibly constant considering the small number of

TABLE IV.

Table showing the Percentage of Persons attacked by Smallpox, with the different numbers of Vaccinal Cicatrices at each age period, in the Epidemics 1870—72, 1880—85, 1892—95, *and* 1902 *in London.*

Group I. Persons 0—10 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	0—10	24·6	25·5	22·6	27·3
1880—85	10—20	20·7	25·7	31·3	22·2
1892—95	20—30	14·3	23·7	28·3	35·1
1902	30—40	7·9	14·2	24·8	51·1

Group II. Persons 10—20 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	10—20	22·2	36·8	23·3	17·5
1880—85	20—30	21·3	32·3	26·0	19·9
1892—95	30—40	20	27·0	29·0	23
1902	40—50	18·4	28·4	27·1	25·9

Group III. Persons 20—30 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	20—30	24·6	39·6	19·4	16·7
1880—85	30—40	—	—	—	—
1892—95	40—50	28·6	28·4	15	22·8
1902	50—60	16·7	35·6	23	22·5

Group IV. Persons 30—40 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	30—40	29·8	41·6	17·8	16·6
1880—85	40—50	—	—	—	—
1892—95	50—60	24·7	30·3	21·3	23·6
1902	60—70	26·4	31·8	18·7	23·1

Group V. Persons 40—60 years, 1870—72.

Epidemic	Age Periods	One Scar	Two Scars	Three Scars	Four Scars
1870—72	40—60	32·7	39·1	16·6	10·4
1880—85	50—70	—	—	—	—
1892—95	60—80	50	32·3	5·9	11·8
1902	70—	39·9	37·1	11·2	11·2

attacks at these ages. Persons with four marks are relatively well protected under five years, which is to a certain extent explained on the ground that the bulk of the patients at that age period belong to the class with very imperfect marks. Between twenty and forty years the advantage seems greatest, and after forty it has apparently entirely disappeared. Exactly the opposite would appear to be the case with persons with one mark. The cause of this in the early years, however, has just been explained, and the apparent decrease in later years is accounted for largely by the fact that a steady equalisation of susceptibility among the four groups takes place, and the number of persons with several cicatrices increases in proportion. It is to be noted that it is apparently between twenty and forty that persons with four scars have most advantage over persons with one. It may be added that the group of those with three marks resembles in general that with four, and the group with two that with one. This question of relative susceptibility at different ages is closely allied to that just discussed regarding the lowering of the mean age in those groups where vaccination is most perfectly performed. It certainly cannot be inferred from the latter that the more incomplete the vaccinal protection the more likely is the attack of smallpox to be delayed, provided such an attack is going to come. It appears much more likely that the decay of protective power is longer delayed, and that consequently while for the first twenty years or so any kind of fair mark indicates much the same protective power, there is a period in the middle ages of life when those who have good cicatrices are much better protected than those with inferior, and again after this period of relative protection has decayed all again become equally susceptible*. The only statistics which are available to check this conclusion are those of Warrington in the epidemic 1892–3, where a census as to the state of

* How this produces a higher mean age among those badly vaccinated than among those well vaccinated can be seen if the figures of Table V. are examined, though it is perhaps not immediately apparent. Here the divisions do not lend themselves to any accurate calculation of mean ages. In the approximations however the mean age of all those found to be badly vaccinated is in much the same excess over that of all found to be well vaccinated, as the mean age of those in the former class who were attacked is in excess of the mean age of those in the latter. But this high mean age among the badly vaccinated attacked by smallpox is not necessarily a consequence of this grouping of the population in the houses invaded by smallpox. As high ages are more susceptible among the badly vaccinated the population of the houses invaded will necessarily contain more old persons as there must be some correlation of age among persons living in the same houses, young families being of course more common among young parents. The high mean age referred to is independent of this, as will be seen if the percentage attacks in both are calculated. These show that in equal populations the mean age of those badly vaccinated attacked by smallpox would be in considerable excess of that of the well vaccinated.

Proportions Attacked.

Age Periods	Well Vaccinated	Badly Vaccinated
0—10	3·7	6·8
10—20	22·6	19·5
20—30	48·3	47·1
30—40	27·5	54·9
40—50	16·2	19·7
50—	13·4	19·5

vaccination was taken for the different age periods of all those in infected houses. Unfortunately the vaccinal scars are simply divided into two classes according as they exhibit good or imperfect characters. It will be seen that under ten years those well vaccinated have a certain measure of protection probably for the reason

TABLE V.

Correlation between Exposed and Attacked, Vaccinated Well and Badly, Warrington, 1893.

	Well Vaccinated		Badly Vaccinated		
Age Periods	Not Attacked	Attacked	Not Attacked	Attacked	Correlation
0—10	466	18	124	9	+·21
10—20	346	114	136	32	−·13
20—30	144	135	74	66	−·03
30—40	95	36	46	56	+·43
40—50	98	19	61	15	+·09
50—	45	7	68	14	+·14

already indicated, from ten to thirty years there is practically no difference between the classes. Between the ages of thirty and forty the advantage however is very definitely in favour of those with good marks, and after the age of forty this is still present, though not nearly so marked. These figures so far as they go confirm the explanation just given.

Before proceeding further to discuss the character of the immunity conferred by vaccination it is necessary to have as far as possible an accurate knowledge of the susceptibility to smallpox exhibited at different ages among persons who have no acquired protection at all either from vaccination or previous attack of the disease. With regard to the age susceptibility to smallpox in an unvaccinated community not a great deal can be said. In prevaccination days statistics were rarely kept with any accuracy, and though records of death in age periods exist relating to considerable numbers of persons and considerable periods of time, yet these afford a quite inadequate basis on which to compute the susceptibility at different ages. The most complete tables calculated from the number of deaths are those published by Duvillard with the authority of the French Academy of Sciences in 1806. He deduces the result that persons at all ages are about equally susceptible to smallpox, but that the maximum susceptibility is between the ages of ten and eleven years, though the excess is small. His conclusions lead him, however, into the difficulty of requiring that a much larger number of persons at a higher age should be attacked by smallpox than is actually observed. He explains this on the ground that the great majority of these attacks are so mild as to escape notice. Now of this there was no evidence at the time at which Duvillard wrote, nor is there any at the present date, when the diagnosis of modified smallpox has been brought to as great a pitch of perfection as any clinical diagnosis can. Consequently his conclusions afford an example of the untrustworthiness of calculating the susceptibility at different ages from the number of deaths alone without any knowledge of the actual case mortalities. For the determination of the susceptibility at different ages among persons who

are unvaccinated at the present day certain statistics exist. The chief of these are contained in the account of the epidemic in Sheffield in 1887 presented in Dr Barry's report to the Local Government Board. In this case a census was taken of nearly the whole population. The facts are tabulated so as to give the number of cases at each period who were unvaccinated, vaccinated, or revaccinated, and also those who had previously had smallpox. The state as regards the vaccination of the inhabitants of those houses actually invaded by smallpox is also tabulated separately. On account of the size of the epidemic this census must be held as the most important group of statistics at our disposal. It seems better to consider fully in the first instance the conclusions which may be based on these figures, and afterwards to compare the results with the statistics affecting naturally much smaller numbers which have been published with regard to other towns.

Considering, in the first place, the alterations in susceptibility to smallpox among the unvaccinated which takes place as the age increases, and expressing the relationship in terms of the correlation coefficients between those exposed to infection who escape, and those exposed who are attacked for each two succeeding age periods, it is found that these coefficients vary in the following

TABLE VI.

Correlation between those who were Attacked, and those who Escaped in Vaccinated and Unvaccinated respectively, at each age and that immediately succeeding.—Sheffield, 1887—8.

Age Periods	Vaccinated	Unvaccinated	Age Periods	Vaccinated	Age Periods	Unvaccinated
0— 1 & *1— 5*	−·10	−·28	—	—	*0— 1* & *1— 5*	−·28
1— 5 & *5—10*	−·21	−·17	—	—	*1— 5* & *5—10*	−·47
5—10 & *10—15*	−·34	−·53	—	—	*5—10* & *10—15*	−·84
10—15 & *15—20*	−·29	·72	—	—	*10—15* & *15—20*	−·20
15—20 & *20—25*	·04	·04	*15—20* & *20—25*	·04	*15—20* & *20—25*	−·18
20—25 & *25—30*	·01	·38	*20—25* & *25—30*	·04	*20—25* & *25—30*	·24
25—30 & *30—35*	·09	·14	*25—30* & *30—35*	·13	*25—30* & *30—35*	·34
30—35 & *35—40*	·26	·24	*30—35* & *35—40*	·31	*30—35* & *35—40*	·54
35—40 & *40—45*	·47	·75	*35—40* & *40—45*	·53	*35—40* & *40—45*	·73
40—45 & *45—50*	·48	−·25	*40—45* & *45—50*	·67	*40—45* & *45—50*	·47
45—50 & *50—55*	·01	·80	*45—50* & *50—55*	·68	*45—50* & *50—55*	·95
50—55 & *55—60*	·15	−·17	*50—55* & *55—60*	·77	*50—55* & *55—60*	·93
55—60 & *60—*	·02	·04	*55—60* & *60—*	·79	*55—60* & *60—*	·92

way. From birth till between ten and fifteen years they are negative, indicating that the susceptibility steadily increases to this age, as was before found by Duvillard, and that thereafter each succeeding coefficient is positive of greater or less amount. If the susceptibility under one year is chosen for comparison with that of each succeeding age period it is seen that the strength of resistance which was less marked between ten and fifteen years has between twenty and thirty again increased so as to be equivalent to that in the first year of life, and thereafter steadily becomes greater. The period of maximum susceptibility between ten and fifteen years is one which is well marked in a large number of epidemics, although incapable of any very accurate measurement. The number

of children successfully vaccinated is published as a Government Return for every year since 1872, so that for epidemics occurring since 1890 for any definite place

TABLE VII.

Table showing the rate per Million of Attack by Smallpox among the Unvaccinated calculated as described in the text with the number of Attacks in an epidemic of Smallpox in 1726 *at Aynho in Northamptonshire for comparison.*

Age Periods	London, 1902	Leicester, 1892–93	Gloucester, 1895–96	Aynho, 1726
0— 5	—	255	727	13
5—10	315	607	909	15
10—15	543	700	955	33
15—20	584	554	774	14
20—25	339	396	434	16
25—30	225	386	446	9
30—35	150	77	351	32*

a rough approximation can be made of the respective number of vaccinated and unvaccinated, it being always borne in mind that the number of unvaccinated thus calculated is a maximum, and that many persons included have probably been vaccinated at some period subsequently during the existence of smallpox scares. The occurrence of such vaccination obviously becomes more probable the older the patient is, so that if a maximum is found between ten and fifteen years it is probably, as thus measured, to be in defect of the reality. The relative insusceptibility of old persons is also in statistics of epidemics an almost constant feature but the small numbers of unvaccinated persons attacked with smallpox from twenty years upwards makes the tracing of the development of insusceptibility in many cases less definite than in the case of Sheffield. This represents the main course of the increase and the decrease of susceptibility towards smallpox presented by the unvaccinated person. It is in general agreement with the few figures which exist regarding the age incidence of smallpox in the eighteenth century. This age incidence of course depends in a very large measure in any locality on the frequency with which epidemics occur. If epidemics break out every three or four years, as is known to have been the case in many places, then it is obvious that the most frequent age of attack will be that between two and three years, a characteristic well shown for example in the statistics of some German towns. The ages of two to three years indeed seem to constitute a period of greater relative susceptibility, as is already indicated in the figures for Sheffield, where the correlation coefficient between the age periods 0–1 and 1–5 years is greater than that between the periods of 1–5 and 5–10, though the numbers are too small to make this absolute. If, however, epidemics occur at greater intervals the natural period of greatest susceptibility will tend to become more evident. A maximum between ten and fifteen years is shown markedly in the statistics of Aynho in Northamptonshire, given by Dr Jurin, and, though in his record there is no statement to indicate the intervals between the succeeding epidemics, it is probable that this represents an epidemic outbreak of smallpox among a comparatively virgin community.

* This is number of cases above 30.

Still confining our consideration to the Sheffield figures we proceed to investigate the susceptibility among the vaccinated. Among these, as we pass from age period to age period an increase of susceptibility occurs from the period of vaccination till between the ages of fifteen to twenty years. At this age the period of greatest susceptibility is reached, and thereafter immunity is gradually regained, increasing with succeeding years, but never actually reaching that existing immediately after the operation was performed. It is to be noticed that the growth of immunity with old age proceeds from the age of twenty upwards in practically the same manner as among the unvaccinated. The curve is much more continuous, as is to be expected from the larger numbers which are available for its construction. The correlation coefficients fall slightly short of the similar ones for the unvaccinated, which is to be expected, as the old age protection starts originally from a higher level, the acquired immunity not being wholly lost at the age chosen for comparison. It is a matter of great interest from the point of view of the theory of immunity that with this large mass of statistics suitable for the calculation of the effect which one form of immunity produces upon another, namely, the effect of the presence of acquired immunity upon the growth of natural immunity, it is found that the latter develops exactly as if the former were completely absent. This is a result which could not have been anticipated, and its meaning will be further discussed in another paper.

The relative susceptibilities of the vaccinated and unvaccinated now falls to be examined, and here the Sheffield census is exceptionally valuable, as it allows the relative susceptibilities of vaccinated and unvaccinated to be determined and

TABLE VIII.

Table showing Correlation between Vaccinated and Unvaccinated Persons as regards Attack by Smallpox and escape from Attack when the numbers are considered of all persons (a) in the town itself, (b) in infected houses.

	(*a*) All persons in the town considered	(*b*) Those in the infected houses alone considered		
Age Periods	Sheffield, 1887–88	Sheffield, 1887–88	Dewsbury, 1892*	Leicester, 1892–93
0— 1	·84	·94	·90	1
1— 5	·63	·94	·93	1
5—10	·63	·96	·84	·93
10—15	·67	·99	·56	·77
15—20	·45	·72	·73	·50
20—25	·34	·74	·66	
25—30	·12	·40		
30—35	·17	·35	—	·17
35—40	·16	·39		
40—45	·11	·28	·30	
45—50	·19	·49		
50—55	−·05	−·11	−·03	
55—60	·05	·07		
60—	·11	·05		

* In this instance the number of revaccinated persons and of those who have had previous attacks of smallpox are excluded in the statistics, so that the correlation coefficients are as free from error due to the admittance of such persons as is possible. The whole numbers affected are however small.

compared, not only when the risk of infection is comparatively slight, as in the general population, but when it is much more considerable, as in infected houses. In the annexed table are given in parallel columns a series of coefficients expressing the relative susceptibilities under each of these conditions for all age periods. It will be seen that the protection which vaccination affords against attack is much less strong, considered in relation to the total numbers of such persons in the town generally, than to the numbers in houses actually infected. In the former it ceases to be considerable after twenty-five years of age, and is practically negligible after the age of fifty. In infected houses, however, this decline is not nearly so marked, but after the age of fifty it also becomes practically negligible. This is an example of a property which at times seems to ensure a greater value to action on a mass than on a small section of that mass which differs from the mass in being apparently more vulnerable. It is not a case of the poison acting equally on the two classes as may be taken for granted is likely to be the case in invaded houses, but that the relative sparseness of unvaccinated persons affords them a protection beyond what random selection would give. It is conceivable that some such factor plays an adjuvant part in natural selection. There are no other statistics which enable comparison to be made between those in infected houses and those of the general population, but the relative susceptibility between the two classes of vaccinated and unvaccinated in infected houses has been calculated for some other epidemics, Leicester, etc., which show local differences, but exhibit on the whole the same kind of relation from age period to age period which has been noted in the case of Sheffield. All the figures agree in making a rapid fall in the protective power of vaccination after the age of fifteen years. It is, however, to be noticed that the protection against a severe attack of smallpox or death afforded by vaccination is of a much more permanent nature than that against an attack of the disease. This is to be expected when the clinical resemblance between vaccination and the second stage of the course of smallpox both as regards local appearances and the course of the pyrexia is recalled, and when it is noted that the clinical course of smallpox suggests the presence of two very different classes of toxins in the two different stages of the disease, so that the immunity against the first stage is much more readily lost than that against the second. That this is the explanation will be apparent when the close relationship between the protection against severe attack, that is an attack with a severe second stage of the fever, and a fatal attack is noted. There are no very ample data to determine this comparison, but in the annexed table will be seen in parallel columns the correlation betwixt recoveries and deaths, and mild cases and severe for the Gloucester epidemic of 1896, and it will be seen that the correspondence, considering the small numbers involved, is sufficiently close. The figures which Macdonell has already given are quoted for comparison, when it will be readily seen that the deaths give a very fair measure indeed of the severity of the disease, closer than might be expected when it is considered how large a factor the personal equation plays in the tabulation of degrees of severity. This is important, as the great majority of statistics

as published refer only to cases and deaths, and give no detail of the character of the illness.

TABLE IX.

Correlation, Vaccinated and Unvaccinated. Recoveries and Deaths, and Cases Mild and Severe, for Epidemic in Gloucester, 1896.

Age Periods	Recoveries and Deaths	Cases Mild and Severe
0— 5	—	—
5—10	—	·94
10—15	·94	·93
15—20	·77	·75
20—30	·63	·88
30—40	·73 } ·63	·68
40—50	·43 }	·63
50—60	} ·63	·60
60—	}	—

TABLE IX *bis.*

Correlation between Cases Mild and Severe and Recoveries and Deaths. Vaccinated and Unvaccinated (Macdonell).

	Recoveries and Deaths	Mild and Severe
Sheffield	·77	·79
Glasgow, 1892—5 ...	·78	·91
Glasgow, 1901 ...	·63	·51

The way in which the protection against death from smallpox afforded by vaccination is gradually lost is of considerable interest. As shown in the appended table it gradually decreases from the date of vaccination up to the age of twenty-

TABLE X.

Correlation, Vaccinated and Unvaccinated, Recoveries and Deaths in Various Epidemics.

Age Periods	London 1856–61	London 1870–72	Sheffield 1887	Gloucester 1896	London 1902
0— 5	—	—	1	1	1
5—10	·34	·74	·79	1	·77
10—15	·65 }	} ·60	·78	·94	·93
15—20	·59 }	}	·76	·77	·74
20—25	·66 }	} ·59	·67	} ·63	·61
25—30	·65 }	}	·49	}	·67
30—35 }	}	}	·55	}	·51
35—40 } ·70	} ·46	}	·53	} ·63	·43
40—45 }	}	}	·38	}	}
45—50 } ·76	} ·31	}	·64	}	} ·74
50—55	}	} ·42	·48	}	}
55—60	}	}	} ·49	} ·62	} ·65
60—65	}	}	}	}	}
65—70	}	}	}	}	} ·46

five, after which it is moderately constant. One point, however, of interest becomes manifest here, and that is the way in which the correlation coefficients for the different ages between vaccinated and unvaccinated tend to assume higher values, at the ages of 10 to 20 years*, as the century has gone on. Taking the first set of statistics which exist, namely those of Dr Marson, passing from these to the great epidemic of 1870–72 in London up to the last epidemic of 1902, we see the tendency for those coefficients to become larger continuously manifest, and the corresponding figures for the other epidemics for which they have been calculated seem to fit into place much as might be expected from their date of occurrence. This affords some confirmation of what has been suggested earlier in this paper in connection with the rise in age at which vaccinated persons are attacked with smallpox, namely, that vaccinal lymph has gradually gained potency through being passed for many generations through the human subject †.

The question now arises as to whether in different epidemics the extra protection which good scars are known to afford to their possessors compared with bad scars assumes greater relative importance in mild or severe epidemics. There seems, however, to be little difference. In the accompanying table will be seen

TABLE XI.

Correlation between Recoveries and Deaths in the Epidemics 1902 *and* 1892—95, *and* 1902 *and* 1870—83 *for each Vaccinal Group.*

Nature of Group	1902 & 1892–95	1902 & 1870–83
One Scar	·22	·43
Two Scars	·18	·37
Three Scars	·24	·43
Four Scars	·21	·52

the correlation between cases and deaths for the epidemic in 1902 and those in 1892–95 and 1870–83. These coefficients indicate that there is no great difference in a severe epidemic or in a mild between the manner in which each different group is affected in comparison with the same group in a severe or in a mild epidemic. The figures, it is true, show some little difference, but not much when it is considered that the change in the age distribution of the different groups has been proceeding not by any means equally in them all.

Another point which may be noted is the manner in which the relative protection against death between those with good and those with bad marks varies. It is found to differ not very considerably below the age of twenty, to

* The whole correlation coefficients are given in the table, but only the early years are referred to in the text, as adult primary vaccination was very common in the first half of the century.

† It is this change in the correlation coefficients which serves to differentiate the two interpretations which one puts on the rise of the mean age: (1) that due to the lymph gaining potency which is the cause of the rise of the general mean age of vaccinated persons attacked by smallpox, here the correlation between vaccinated and unvaccinated has increased as the century has gone on: and (2) the higher mean age of indifferently vaccinated persons taking smallpox than of well vaccinated, here the correlation between recoveries and deaths is in favour of the well vaccinated. It is thus seen that the mean age may be raised in two different ways. (Cf. footnote, p. 319.)

be at a maximum between twenty and thirty years, and then to gradually disappear towards old age. The figures here, however, are not large enough to base any definite conclusions.

It now falls to be considered at what rate immunity develops after vaccination, that is to say, how long must vaccination be performed prior to the onset of smallpox to prevent the attack? The incubation period of smallpox is from twelve to fourteen days, and it is found to be practically an absolute rule that no person develops smallpox more than fourteen days after a successful vaccination has been performed. If vaccination be performed during incubation period of smallpox the attack is seen often to be much modified. Here again the statistics refer to small numbers. In the first place, taking all ages together and estimating the correlation between the attacks and deaths in unvaccinated persons and in

TABLE XII.

Correlation between Recoveries and Deaths in Unvaccinated Persons, and those Vaccinated at various Dates prior to onset of illness.

Time before onset	Gloucester (all ages)	London (all ages)	London (ages above 1 year)
Days	1896	1902	1902
0— 3	·13	·06	·40
3— 6	·16	·12	·40
6— 9	·65	·47	·72
9—14	·14	·12	·45
14—	1·00	1·00	1·00

those vaccinated after infection, we find that even among those vaccinated within three days of the onset of the illness there is a distinct positive correlation in favour of the process. Among those vaccinated from three to six days before the onset of attack the correlation is slightly higher, from six to nine days it is very considerable, from nine to fourteen days, however, it is again very much less, apparently due to the fact, as will be shown later, that only a few persons who are really very susceptible develop the disease after nine days. After fourteen days the protection against death is absolute. If children under one year be eliminated from the numbers considered the protection is seen to be considerably higher, but still to exhibit a maximum at the period of six to nine days.

Though the subtraction of those cases under one year makes such a difference in the correlation coefficients, yet when those who have been vaccinated more than six days before the onset of the illness are compared for the varying age

TABLE XIII.

Correlation between Recoveries and Deaths of those Unvaccinated, and those Vaccinated 6—14 *days prior to onset of illness. London,* 1902.

Age Periods		
0— 1	·61	
1— 5	·63	
5—10	·73	·60
10—15	·38	
15—20	·49	
20—30	·61	

periods with those who are unvaccinated at the same age period it is found that the correlation is practically constant from birth onwards, and that the smaller correlation coefficients obtained when all ages are grouped together is brought about by the fact that a large proportion of the total cases are under one year, and that among these there is so special a mortality as to vitiate seriously the value of the correlation coefficients of the statistics when all ages are combined.

As to the actual susceptibility of persons vaccinated after exposure to smallpox there are no actual figures in existence. Some, however, can be inferred in the Gloucester epidemic; out of 653 persons vaccinated after exposure only 85 developed the disease as compared with 604 out of 832 among the unvaccinated who were exposed to infection and were not vaccinated. Out of the 85, 23 were vaccinated on the day of onset of the disease or on the day after, at a time when, therefore, the process could have no preventive action. The distribution of the remaining 62 persons is as follows: The number of those vaccinated from 0–3 days before the onset of attack was 19, from 3–6 days also 19, from 6–9 days 18, while for 9 days and upwards only 6. Now the great majority of persons exposed to infection are vaccinated within four days of the exposure, so that it becomes evident that if the operation be performed up to the ninth day before that on which onset of the attack could be expected there is a very high probability that no further developments will take place. This is a question on which statistics cannot be compiled, because, though a person may be exposed to infection for three to four days there is nothing to indicate on which day he may be infected, if at all, unless he subsequently develops the disease, so that the above figures are probably as satisfactory as any can be. The similar relation which exists between persons revaccinated more than six days before the onset of the

TABLE XIV.

Correlation between Recoveries and Deaths of those Vaccinated in Infancy only, and those Revaccinated 6—14 *days prior to onset of illness.*

Age Periods	London, 1902
20—40	·46
40—50	·35

illness shows a correlation of a similar kind, but not so great as in the former class, which is to be expected as before remarked, as the level of the protection from which the improvement is made is originally much higher. The smallness of the figures does not permit it to be given for more than two age groups.

The effect which an attack of smallpox exercises in establishing immunity is much more difficult to ascertain than that produced by vaccination. In general the great bulk of persons who are vaccinated have been vaccinated in infancy. With regard, however, to those who have had smallpox there are no data as to the ages at which they were attacked. In London between 1836 and 1851 out of 2654 unvaccinated persons suffering from smallpox treated in hospital only

690 were under the age of ten years; between 1870 and 1883 out of 2169 like persons 1197 were under the age of ten years. In both of these sets of statistics the distribution of the numbers from birth till ten years is fairly uniform. So that it may be taken for granted that towards the end of last century among persons over twenty years of age who had had smallpox previously one half was the greatest proportion of these who had the attack during the first ten years of life. In the accompanying table I have collected the details of fifty-one cases

TABLE XV.

Table of Second Attack of Smallpox.

Age at Second Attack.

Age at First Attack.	*15—20*	*20—25*	*25—30*	*30—35*	*35—40*	*40—45*	*45—50*	*50—55*	*55—60*	*60—65*	*65—70*	*70—75*	*75—80*	*90—*	Totals
0— 5	4	5	2	4	4	1	2	2	1	1	2	1	—	1	30
5—10	—	4	—	1	—	—	2	1	—	1	—	—	—	—	9
10—15	—	—	1	1	2	—	1	1	—	—	—	—	—	—	6
15—20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20—25	—	—	—	1	—	1	—	—	—	—	1	—	—	—	3
25—30	—	—	—	—	—	1	—	1	—	—	—	—	—	—	2
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
40—45	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	4	9	3	7	6	3	5	6	1	2	3	1	—	1	51

where persons have had a second attack of smallpox between the years 1887 and 1902, and of these fifty-one it is seen at once that thirty suffered from their first attack of smallpox below the age of five years, nine between the age of five and ten years, leaving a total of only twelve who had their first attack of smallpox above this age. This would seem to show that the protection afforded by an attack of smallpox in childhood is much less complete than that afforded by an attack after ten years. A study of the figures in the table seems to show that this is not due simply to the natural lessening in numbers which occurs in the population at each age period from childhood onwards. A certain amount of support is lent by this to the suggestion advanced earlier in this paper that vaccination performed in early adult life afforded a longer period of protection than when performed in infancy, and in both instances it may be surmised that as the natural susceptibility to smallpox increases from birth onwards till about twelve years there may be a greater loss of acquired immunity during the same period of life than is likely to occur when the natural protection against smallpox has become more definitely established. With regard to the protection which an attack of smallpox provides against death even less can be said. Of these fifty-one cases seven died. Five of these deaths, however, occurred in one epidemic out of twenty-three cases, and, although one of these five died from apoplexy, yet even four deaths out of twenty-three cases represents a mortality so much in

excess of what might be expected as to make it probable that these figures are not sufficiently large to be taken as trustworthy. The actual protection, however, which one attack of smallpox affords against a second is much larger than that conferred by vaccination; this protection, judging from the Sheffield census, is very high, the correlation coefficient being ·6, and this high correlation of protection against attack suggests the probability of some error in the death rate as given above.

The protection given by revaccination is still higher than that given by a previous attack of smallpox and by recent revaccination practically absolute. Of the 400,000 persons revaccinated in Glasgow in 1901 six were admitted during the epidemic of 1904; three of these stated that there had been no local reaction, they were therefore not effectively revaccinated; one stated that there had been a local reaction, which, however, left no scar. In the other two the condition of the arm indicated successful revaccination. That is out of 400,000 persons three at the most were affected with smallpox after a lapse of three years, while of the other 250,000 inhabitants of Glasgow more than 800 succumbed. In the statistics published in the London epidemic of 1902 the mean period which has elapsed between revaccination and the attack of smallpox (23·9 years) falls considerably short of the mean age of those attacked by smallpox in the same epidemic (28·6 years) with the protection alone of infantile vaccination. These persons, however, must represent a more susceptible class, for, judging from the Sheffield epidemic, the correlation between the revaccinated who escape and those who were attacked and the like classes of the vaccinated is very considerably in favour of the former.

TABLE XVI.

Table of Revaccinated Persons Attacked by Smallpox. London, 1902.

Age at Attack.

Age at Revaccination.	*10—15*	*15—20*	*20—25*	*25—30*	*30—35*	*35—40*	*40—45*	*45—50*	*50—55*	*55—60*	*60—65*	*65—70*	*70—75*	Totals
0— 5	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5—10	1	—	—	1	1	1	—	4	1	—	—	—	—	9
10—15	1	—	2	5	2	4	7	5	—	—	1	—	—	27
15—20	—	—	1	—	6	2	10	3	2	2	—	1	—	27
20—25	—	—	—	—	1	2	5	1	2	—	—	1	—	12
25—30	—	—	—	—	1	—	1	2	—	1	—	—	—	5
30—35	—	—	—	—	—	—	1	3	2	—	—	1	—	7
35—40	—	—	—	—	—	—	—	—	1	—	—	—	—	1
40—45	—	—	—	—	—	—	—	—	1	—	—	—	—	1
45—50	—	—	—	—	—	—	—	—	—	1	—	—	—	1
50—55	—	—	—	—	—	—	—	—	1	—	—	—	—	1
55—60	—	—	—	—	—	—	—	—	—	—	—	—	—	—
60—65	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	—	3	6	11	9	24	18	10	4	1	3	1	92

In the accompanying table those cases of revaccinated persons attacked by smallpox during the London epidemic of 1902 are given; the number is ninety-two in all, and among these two deaths occurred, in both of which the period of the latest vaccination had been at least twenty-four years previously. Few inferences can be drawn from this table; it is, however, to be noted that more than a third of the cases occur in persons revaccinated before the age of fifteen years, who, as far as the statistics of this hospital go, constitute a very much smaller proportion of the total revaccinated persons, indicating again that the operation performed at this age affords less permanent protection than when done later.

In conclusion, it is necessary to state that all the correlation coefficients in this paper were calculated by the short method given at the end of Prof. Pearson's memoir on the correlation among attributes not quantitatively measurable * by means of the formula

$$r = \sin\frac{\pi}{2}\left\{\frac{1}{\sqrt{1+\kappa^2}}\right\}, \text{ where } \kappa^2 = \frac{4abcdN^2}{(ad-bc)^2(a+d)(b+c)}.$$

If the values of r corresponding to $\log\kappa^2$ from $\bar{1}$ to 4 be first calculated, as is easily done with the help of a table of Gaussian addition logarithms, then it is a matter of only a few minutes to obtain a correlation coefficient. Prof. Pearson has found this formula true in most instances within the probable error; I myself have checked it with about twenty examples and only once found it give a result outside that error, so for the accuracy required in this paper it may be considered sufficient. When the numbers are in nearly all cases small the probable error is large, and little stress can be placed on the accuracy of an individual coefficient. A far better test in a subject like this, when very large numbers of coefficients have to be calculated, is their general concordance. In many cases they form part of a series which is *à priori* continuous, and consequently each affords a mutual check on the other of much greater value considering the number of cases involved than the calculation of the probable error. Considering the series tabulated in this paper, many instances will be seen where the coefficients fall so out of line with their immediate neighbours that it is evident that some large error is present.

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ON TESTS FOR LINEARITY OF REGRESSION IN FREQUENCY DISTRIBUTIONS.

By JOHN BLAKEMAN, M.Sc., B.A.

In considering any frequency distribution of two characters, say x and y, one of the things to be determined is whether the regression curve of y on x is linear or non-linear.

In his memoir "On the general theory of skew correlation and non-linear regression*" Karl Pearson has given the test for linearity of regression expressed in terms of the correlation coefficient r and the correlation ratio η.

It is there shown that, while η is by definition an essentially positive quantity r may be either positive or negative; that η^2 and r^2 are both quantities which can only vary between the limits 0 and 1 and moreover η^2 is never less than r^2; finally, that the necessary and sufficient condition for linear regression is $\eta^2 - r^2 = 0$.

It has been remarked that r may be positive or negative, but it is obvious that whenever r works out negative we can, by changing the direction of either one of the axes of reference, make r positive. Thus, in all frequency distributions, the axes may be so drawn that r shall be positive. There is therefore no loss of generality in assuming r positive and, for simplicity of statement, in what follows r will always be taken as positive.

With this convention we see that, if we write $\zeta = \eta^2 - r^2$, $\Im = \eta - r$, $\varpi = \log_e \frac{\eta}{r}$, the test for linear regression may be written indiscriminately as $\zeta = 0$, or $\Im = 0$, or $\varpi = 0$ and in many other equivalent ways.

Now, in any actual distribution, we shall not expect to find the quantities ζ, $\Im$, ϖ absolutely zero but, even when the regression of y on x is linear, we shall expect the values of ζ, $\Im$, ϖ to be influenced in the usual way by the fact that our material is only a random sample of the general population.

Thus, as in all statistical tests, having found ζ, or $\Im$, or ϖ, to have certain positive values, we want to know if these values are such as might arise from

* *Drapers' Company Research Memoirs.* Biometric Series II. Dulau and Co.

errors due to random sampling from a general population whose regression is linear, or whether the values are such as to indicate significant non-linearity.

For this purpose we compare the calculated value of ζ, or ϑ, or ϖ with its probable error, where, if the standard deviations of ζ, ϑ, ϖ are written Σ_ζ, Σ_ϑ, Σ_ϖ the probable errors, say E_ζ, E_ϑ, E_ϖ, are defined by the equation $E = \cdot 67449\Sigma$.

While Karl Pearson has given in his memoir a formula for Σ_η, and the formula for Σ_r is well-known, no formula has yet been given for calculating any such quantity as Σ_ζ, Σ_ϑ, Σ_ϖ for the testing of linear regression.

It is the object of the present investigation to determine such formulae; the obvious thing to do would be to determine Σ_ζ, as ζ is the quantity occurring naturally in Karl Pearson's work; but the formula for Σ_ϖ is so very much simpler that it has been made the basis of this investigation; it is then shown how to determine Σ_ζ and Σ_ϑ from Σ_ϖ and other known quantities.

Thus it is at the worker's choice to test linearity of regression by comparing any one of the quantities ζ, ϑ, ϖ with its probable error.

Following Karl Pearson's notation n_{x_ρ} will denote the number of individuals in the x_ρ-array of y's, n_{y_s} the number of individuals in the y_s-array of x's and $n_{x_\rho y_s}$ the number of individuals in the sub-group common to these two arrays. N is the total number of individuals in the table; $\bar{x}$, $\bar{y}$ are the means of the two characters, while σ_x, σ_y are their standard deviations; y_{x_ρ} is the mean of the x_ρ-array of y's while $\sigma_{n_{x_\rho}}$ is the standard deviation of that array and $p_{uv}, \lambda_m, \sigma_M, \eta, r$ are quantities defined by the following equations, S' denoting a double summation:

$$Np_{uv} = S'\{n_{x_\rho y_s}(x_\rho - \bar{x})^u (y_s - \bar{y})^v\},$$

$$N\lambda_m = S\{n_{x_\rho}(y_{x_\rho} - \bar{y})^m\},$$

$$N\sigma_M{}^2 = S\{n_{x_\rho}(y_{x_\rho} - \bar{y})^2\} \quad \{\text{i.e. } \sigma_M{}^2 = \lambda_2\},$$

$$\eta = \frac{\sigma_M}{\sigma_y}, \qquad r = \frac{p_{11}}{\sigma_x \sigma_y},$$

and further we shall define a quantity λ_{uv} by the equation

$$N\lambda_{uv} = S'\{n_{x_\rho}(x_\rho - \bar{x})^u (y_{x_\rho} - \bar{y})^v\}.$$

The fundamental results on which our work is based may be stated as follows:—

Let n, n' be the number of individuals falling within any two mutually exclusive groups, then if Σ_n, $\Sigma_{n'}$, be the standard deviations of n, n' and $R_{nn'}$ the correlation between deviations in n and n' due to random sampling, we have

$$\Sigma_n{}^2 = n\left(1 - \frac{n}{N}\right), \qquad \Sigma_{n'}{}^2 = n'\left(1 - \frac{n'}{N}\right) \quad \text{.....(i),}$$

$$\Sigma_n \Sigma_{n'} R_{nn'} = -\frac{nn'}{N} \quad \text{.....(ii).}$$

General Theorem. Let $n_1, n_2, \ldots$ be the number of individuals in any number of mutually exclusive groups and let quantities X, Y be defined by the equations

$$X = A_1 n_1 + A_2 n_2 + A_3 n_3 + \ldots,$$
$$Y = B_1 n_1 + B_2 n_2 + B_3 n_3 + \ldots,$$

where the A's and B's are quantities which remain constant as the frequency distribution is changed. Then

$$\delta X = A_1 \delta n_1 + A_2 \delta n_2 + A_3 \delta n_3 + \ldots,$$
$$\delta Y = B_1 \delta n_1 + B_2 \delta n_2 + B_3 \delta n_3 + \ldots.$$

Multiply these two expressions together, sum for all random samples and divide by the number of such samples; then using (i) and (ii) we get immediately

$$\Sigma_X \Sigma_Y R_{XY} = -\frac{XY}{N} + A_1 B_1 n_1 + A_2 B_2 n_2 + A_3 B_3 n_3 + \ldots \quad \ldots\ldots\ldots\ldots\text{(iii)}.$$

Problem I. To determine the correlation between the deviations due to random sampling in the values of $n_{x_\rho y_s}$ and $y_{x_{\rho'}}$.

We have
$$n_{x_{\rho'}} y_{x_{\rho'}} = S\{n_{x_{\rho'} y_s} y_s\}.$$

Hence
$$n_{x_{\rho'}} \delta y_{x_{\rho'}} = S\{\delta n_{x_{\rho'} y_s} y_s\} - \delta n_{x_{\rho'}} y_{x_{\rho'}},$$
$$\delta n_{x_\rho y_s} = \delta n_{x_\rho y_s}.$$

Multiply these two expressions together, sum for all random samples and divide by the number of such samples; then

(α) ρ and ρ' different

$$n_{x_{\rho'}} \Sigma_{n_{x_\rho y_s}} \Sigma_{y_{x_{\rho'}}} R_{n_{x_\rho y_s} y_{x_{\rho'}}} = -\frac{n_{x_\rho y_s} n_{x_{\rho'}} y_{x_{\rho'}}}{N} + \frac{n_{x_\rho y_s} n_{x_{\rho'}} y_{x_{\rho'}}}{N} = 0 \quad \ldots\ldots\text{(iv)}.$$

(β) ρ equal to ρ'

$$n_{x_{\rho'}} \Sigma_{n_{x_\rho y_s}} \Sigma_{y_{x_{\rho'}}} R_{n_{x_\rho y_s} y_{x_\rho}} = n_{x_\rho y_s} y_s - n_{x_\rho y_s} y_{x_\rho} = n_{x_\rho y_s} (y_s - y_{x_{\rho'}}) \ldots\ldots\text{(v)}.$$

We shall need, besides, the results proved by Karl Pearson as Propositions III, IV, VIII in the memoir previously cited, which we will state here for reference.

Proposition III.

$$R_{y_{x_\rho} n_{x_\rho}} = 0 \quad \ldots\ldots\ldots\ldots\text{(vi)}.$$

Proposition IV.

$$R_{y_{x_\rho} n_{x_{\rho'}}} = 0 \ (\rho' \text{ not equal to } \rho) \quad \ldots\ldots\ldots\text{(vii)}.$$

Proposition VIII.

$$\Sigma_{\sigma_M{}^2} = \frac{\lambda_4 - \lambda_2{}^2}{4N\lambda_2} + \frac{\chi_1 \sigma_y{}^2 (1 - \eta^2)}{N} \quad \ldots\ldots\ldots\ldots\text{(viii)},$$

where χ_1 is a quantity defined by the equation

$$S\{n_{x_\rho} \sigma_{n_{x_\rho}}{}^2 (y_{x_\rho} - \bar{y})^2\} = N \sigma_y{}^2 (1 - \eta^2) \sigma_M{}^2 \times \chi_1,$$

and is obviously such that, in the case of normal distributions when $\sigma_{n_{x_\rho}}$ is constant and $r = \eta$, we get $\chi_1 = 1$.

Problem II. To find the correlation between the deviations due to random sampling in the values of σ_x and σ_M.

We have $$N\sigma_x^2 = S\{n_{x_\rho}(x_\rho - \bar{x})^2\},$$

$$\therefore\ 2N\sigma_x\delta\sigma_x = S\{\delta n_{x_\rho}(x_\rho - \bar{x})^2\} - 2\delta\bar{x}S\{n_{x_\rho}(x_\rho - \bar{x})\}.$$

But $$S\{n_{x_\rho}(x_\rho - \bar{x})\} = 0.$$

Hence $$2N\sigma_x\delta\sigma_x = S\{\delta n_{x_\rho}(x_\rho - \bar{x})^2\}.$$

Again $$2N\sigma_M\delta\sigma_M = S\{\delta n_{x_\rho}(y_{x_\rho} - \bar{y})^2\} + 2S\{\delta y_{x_\rho} n_{x_\rho}(y_{x_\rho} - \bar{y})\}.$$

Multiply these two expressions together, sum for all random samples, and divide by the number of such samples; then using (iii), (vi) and (vii) we find

$$4N^2\sigma_x\sigma_M\Sigma_{\sigma_x}\Sigma_{\sigma_M}R_{\sigma_x\sigma_M} = -\frac{N^2\sigma_x^2\sigma_M^2}{N} + S\{n_{x_\rho}(x_\rho - \bar{x})^2(y_{x_\rho} - \bar{y})^2\}$$

$$= -N\sigma_x^2\sigma_M^2 + N\lambda_{22}.$$

Hence $$4\sigma_x\sigma_M\Sigma_{\sigma_x}\Sigma_{\sigma_M}R_{\sigma_x\sigma_M} = \frac{\lambda_{22} - \sigma_x^2\lambda_2}{N} \qquad \text{.....(ix).}$$

Problem III. To find the correlation between the deviations due to random sampling in the values of p_{11} and σ_M.

We have $$Np_{11} = S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})\},$$

$$N\delta p_{11} = S'\{\delta n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})\} - \delta\bar{x}S'\{n_{x_\rho y_s}(y_s - \bar{y})\} - \delta\bar{y}S'\{n_{x_\rho y_s}(x_\rho - \bar{x})\}.$$

Hence $$N\delta p_{11} = S'\{\delta n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})\}.$$

But $$2N\sigma_M\delta\sigma_M = S\{\delta n_{x_\rho}(y_{x_\rho} - \bar{y})^2\} + 2S\{\delta y_{x_\rho} n_{x_\rho}(y_{x_\rho} - \bar{y})\}.$$

Multiply these two expressions together, sum for all random samples and divide by the number of such samples; then using (iii), (iv) and (v) we find

$$2N^2\sigma_M\Sigma_{p_{11}}\Sigma_{\sigma_M}R_{p_{11}\sigma_M} = -\frac{N^2p_{11}\sigma_M^2}{N} + S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_{x_\rho} - \bar{y})^2\}$$
$$+ 2S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_s - y_{x_\rho})(y_{x_\rho} - \bar{y})\}.$$

But

$$S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_{x_\rho} - \bar{y})^2\} = S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - y_{x_\rho})(y_{x_\rho} - \bar{y})^2\}$$
$$+ S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_{x_\rho} - \bar{y})^2\},$$

while $$S\{n_{x_\rho y_s}(y_s - y_{x_\rho})\} = 0.$$

Summing with respect to s and keeping ρ constant. Hence

$$S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_{x_\rho} - \bar{y})^2\} = N\lambda_{13}.$$

Again

$$S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - \bar{y})(y_s - y_{x_\rho})(y_{x_\rho} - \bar{y})\} = S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - y_{x_\rho})^2(y_{x_\rho} - \bar{y})\}$$
$$+ S'\{n_{x_\rho y_s}(x_\rho - \bar{x})(y_s - y_{x_\rho})(y_{x_\rho} - \bar{y})^2\}.$$

Sum with respect to s, keeping ρ constant. Since

$$S\{n_{x_\rho y_s}(y_s - y_{x_\rho})\} = 0,$$

and $$S\{n_{x_\rho y_s}(y_s - y_{x_\rho})^2\} = n_{x_\rho}\sigma_{n_{x_\rho}}^2,$$

we get

$$S'\{n_{x_\rho y_s}(x_\rho-\bar{x})(y_s-\bar{y})(y_s-y_{x_\rho})(y_\rho-\bar{y})\}=S\{n_{x_\rho}\sigma_{n_{x_\rho}}{}^2(x_\rho-\bar{x})(y_{x_\rho}-\bar{y})\}.$$

We may write

$$S\{n_{x_\rho}\sigma_{n_{x_\rho}}{}^2(x_\rho-\bar{x})(y_{x_\rho}-\bar{y})\}=N\sigma_y{}^2(1-\eta^2)p_{11}\times\chi_3,$$

where χ_3 is a purely numerical constant, which is obviously equal to unity in the case of normal correlation, when $\sigma_{n_{x_\rho}}{}^2=\sigma_y{}^2(1-r^2)$ for all values of ρ and $\eta=r$.

Hence finally

$$2\sigma_M\Sigma_{p_{11}}\Sigma_{\sigma_M}R_{p_{11}\sigma_M}=\frac{1}{N}\{\lambda_{13}-p_{11}\sigma_M{}^2+2\rho_{11}\sigma_y{}^2(1-\eta^2)\chi_3\} \quad \text{.....(x).}$$

Problem IV. To find the standard deviation of the values of ϖ defined by the equation

$$\varpi=\log_e\frac{\eta}{r}.$$

We have

$$\varpi=\log_e\frac{\eta}{r}.$$

Hence

$$\delta\varpi=\frac{\delta\eta}{\eta}-\frac{\delta r}{r} \quad \text{.....(xi).}$$

But

$$\eta=\frac{\sigma_M}{\sigma_y}.$$

Hence

$$\frac{\delta\eta}{\eta}=\frac{\delta\sigma_M}{\sigma_M}-\frac{\delta\sigma_y}{\sigma_y} \quad \text{.....(xii),}$$

and

$$r=\frac{p_{11}}{\sigma_x\sigma_y},$$

$$\therefore\quad \frac{\delta r}{r}=\frac{\delta p_{11}}{p_{11}}-\frac{\delta\sigma_x}{\sigma_x}-\frac{\delta\sigma_y}{\sigma_y} \quad \text{.....(xiii).}$$

Squaring equations (xi), (xii), and (xiii), and forming the product of equations (xii) and (xiii), then on summing for all random samples and dividing by the number of such samples we get in succession

$$\Sigma_{\varpi}{}^2=\frac{\Sigma_\eta{}^2}{\eta^2}+\frac{\Sigma_r{}^2}{r^2}-\frac{2\Sigma_\eta\Sigma_r R_{\eta r}}{\eta r} \quad \text{.....(xiv),}$$

$$\frac{\Sigma_\eta{}^2}{\eta^2}=\frac{\Sigma_{\sigma_M}{}^2}{\sigma_M{}^2}+\frac{\Sigma_{\sigma_y}{}^2}{\sigma_y{}^2}-\frac{2\Sigma_{\sigma_M}\Sigma_{\sigma_y}R_{\sigma_M\sigma_y}}{\sigma_M\sigma_y} \quad \text{.....(xv),}$$

$$\frac{\Sigma_r{}^2}{r^2}=\frac{\Sigma_{p_{11}}{}^2}{p_{11}{}^2}+\frac{\Sigma_{\sigma_x}{}^2}{\sigma_x{}^2}+\frac{\Sigma_{\sigma_y}{}^2}{\sigma_y{}^2}-\frac{2\Sigma_{p_{11}}\Sigma_{\sigma_x}R_{p_{11}\sigma_x}}{p_{11}\sigma_x}$$
$$-\frac{2\Sigma_{p_{11}}\Sigma_{\sigma_y}R_{p_{11}\sigma_y}}{p_{11}\sigma_y}+\frac{2\Sigma_{\sigma_x}\Sigma_{\sigma_y}R_{\sigma_x\sigma_y}}{\sigma_x\sigma_y} \quad \text{.....(xvi),}$$

$$\frac{\Sigma_\eta\Sigma_r R_{\eta r}}{\eta r}=\frac{\Sigma_{p_{11}}\Sigma_{\sigma_M}R_{p_{11}\sigma_M}}{p_{11}\sigma_M}-\frac{\Sigma_{\sigma_x}\Sigma_{\sigma_M}R_{\sigma_x\sigma_M}}{\sigma_x\sigma_M}-\frac{\Sigma_{\sigma_y}\Sigma_{\sigma_M}R_{\sigma_y\sigma_M}}{\sigma_y\sigma_M}$$
$$-\frac{\Sigma_{\sigma_y}\Sigma_{p_{11}}R_{\sigma_y p_{11}}}{\sigma_y p_{11}}+\frac{\Sigma_{\sigma_x}\Sigma_{\sigma_y}R_{\sigma_x\sigma_y}}{\sigma_x\sigma_y}+\frac{\Sigma_{\sigma_y}{}^2}{\sigma_y{}^2} \quad \text{.....(xvii).}$$

Adding together equations (xiv), (xv), (xvi) and (xvii) multiplied by -2, we get

$$\Sigma_{\varpi}^2 = \frac{\Sigma_{\sigma_M}^2}{\sigma_M^2} + \frac{\Sigma_{p_{11}}^2}{p_{11}^2} + \frac{\Sigma_{\sigma_x}^2}{\sigma_x^2} - \frac{2\Sigma_{p_{11}}\Sigma_{\sigma_x}R_{p_{11}\sigma_x}}{p_{11}\sigma_x} - \frac{2\Sigma_{p_{11}}\Sigma_{\sigma_M}R_{p_{11}\sigma_M}}{p_{11}\sigma_M} + \frac{2\Sigma_{\sigma_x}\Sigma_{\sigma_M}R_{\sigma_x\sigma_M}}{\sigma_x\sigma_M} \quad \text{.....(xviii).}$$

But we have as well-known results

$$\Sigma_{p_{11}}^2 = \frac{p_{22}-p_{11}^2}{N}, \qquad \Sigma_{\sigma_x}^2 = \frac{p_{40}-p_{20}^2}{4p_{20}N} \quad \text{.....(xix),}$$

$$2\Sigma_{\sigma_x}\Sigma_{p_{11}}R_{\sigma_x p_{11}} = \frac{p_{31}-p_{20}p_{11}}{N\sigma_x} \quad \text{.....(xx).}$$

Hence, substituting in equation (xviii) the values given by equations (viii), (ix), (x), (xix) and (xx), we get

$$\Sigma_{\varpi}^2 = \frac{\lambda_4-\lambda_2^2}{4N\lambda_2^2} + \chi_1\frac{1-\eta^2}{N\eta^2} + \frac{p_{22}-p_{11}^2}{Np_{11}^2} + \frac{p_{40}-p_{20}^2}{4Np_{20}^2} - \frac{p_{31}-p_{20}p_{11}}{Np_{20}p_{11}} + \frac{\lambda_{22}-\sigma_x^2\lambda_2}{2N\sigma_x^2\lambda_2} - \frac{1}{Np_{11}\lambda_2}\{\lambda_{13}-p_{11}\lambda_2+2p_{11}\sigma_y^2(1-\eta^2)\chi_3\} \quad \text{.....(xxi),}$$

which may obviously be re-written

$$\Sigma_{\varpi}^2 = \frac{1}{N}\left(\frac{1}{r^2}-\frac{1}{\eta^2}\right) + \frac{p_{40}-3p_{20}^2}{4Np_{20}^2} - \frac{p_{31}-3p_{20}p_{11}}{Np_{20}p_{11}} + \left\{\frac{p_{22}-3p_{11}^2}{Np_{11}^2} - \frac{1-r^2}{Nr^2}\right\} + \frac{\lambda_4-3\lambda_2^2}{4N\lambda_2^2} - \frac{\lambda_{13}-3\lambda_{11}\lambda_2}{N\lambda_{11}\lambda_2} + \frac{\lambda_{22}-3\sigma_x^2\lambda_2}{2N\sigma_x^2\lambda_2} + \frac{1-\eta^2}{N\eta^2}\{(\chi_1-1)-2(\chi_3-1)\} \quad \text{.....(xxii).}$$

This is the complete formula for Σ_{ϖ} to obtain which has been the first object of the present investigation. The formula has been thrown into the form given as equation (xxii) because each term as there exhibited vanishes for a normal distribution, since for normal surfaces we have

$$p_{40} = 3p_{20}^2, \qquad p_{31} = 3p_{20}p_{11}, \qquad \frac{p_{22}-3p_{11}^2}{p_{11}^2} = \frac{1-r^2}{r^2}.$$

Again for a normal distribution

$$y_{x_\rho} - \bar{y} = \frac{r\sigma_y}{\sigma_x}(x_\rho - \bar{x}).$$

Hence

$$N\lambda_4 = S\{n_{x_\rho}(y_{x_\rho}-\bar{y})^4\} = \frac{r^4\sigma_y^4}{\sigma_x^4}.S\{n_{x_\rho}(x_\rho-\bar{x})^4\}$$

$$= \frac{r^4\sigma_y^4}{\sigma_x^4}3N\sigma_x^4 = 3N\lambda_2^2,$$

$$N\lambda_{13} = S\{n_{x_\rho}(x_\rho-\bar{x})(y_{x_\rho}-\bar{y})^3\} = \frac{r^3\sigma_y^3}{\sigma_x^3}.3N\sigma_x^4$$

$$= 3N.r^3\sigma_y^2 \times r\sigma_x\sigma_y = 3N.\lambda_2.\lambda_{11},$$

$$N\lambda_{22} = S\{n_{x_\rho}(x_\rho-\bar{x})^2(y_{x_\rho}-\bar{y})^2\} = \frac{r^2\sigma_y^2}{\sigma_x^2}.3N\sigma_x^4$$

$$= 3Nr^2\sigma_y^2\sigma_x^2 = 3N\lambda_2.\sigma_x^2.$$

Further $\chi_1 = \chi_3 = 1$, while $\eta = r$.

Now the formula for Σ_r^2 consists of a term $\frac{(1-r^2)^2}{N}$ involving r only together with a number of terms involving product moments which are just such terms as the product moment terms of equation (xxii)*. Again the formula for Σ_η^2 consists of a term $\frac{(1-\eta^2)^2}{N}$ involving η only together with a number of terms of exactly the same nature as these moment terms†. In practical statistical work the simple formulae $\Sigma_r^2=\frac{(1-r^2)^2}{N}$, $\Sigma_\eta^2=\frac{(1-\eta^2)^2}{N}$ are used and the nature of the justification for these simple formulae can only be appreciated by a reference to the numerical work of Karl Pearson's memoir where he shows that, as a matter of experience, in actual frequency distributions which are very far from normal, although the separate product terms occurring in the formula for Σ_η^2 may be significant, yet, these product terms occur in such a manner in the formula as to be insignificant in the aggregate. This work suggests that we should consider if anything of a like nature occurs in the case of the formula for Σ_ϖ. Equation (xxii) gives Σ_ϖ^2 as the sum of a term $\frac{1}{N}\left(\frac{1}{r^2}-\frac{1}{\eta^2}\right)$ involving η and r only and a number of product moment terms of exactly the same type as those occurring in the formulae for Σ_r^2, Σ_η^2. I suggest therefore the formula

$$\Sigma_\varpi^2=\frac{1}{N}\left\{\frac{1}{r^2}-\frac{1}{\eta^2}\right\} \quad \text{.....(xxiii),}$$

as being exactly analogous to the simple formulae for Σ_r^2, Σ_η^2; the complete justification for the formula (xxiii) will be found in the practical statistical work given later in the paper.

Before proceeding to the numerical work I will give some formulae which enable us to determine other quantities of importance when Σ_ϖ^2 is known.

Problem V. To find the correlation between the deviations due to random sampling in η and r.

We have
$$\Sigma_\varpi^2=\frac{\Sigma_\eta^2}{\eta^2}+\frac{\Sigma_r^2}{r^2}-\frac{2\Sigma_\eta\Sigma_r R_{\eta r}}{\eta r}$$
$$=\left(\frac{\Sigma_\eta}{\eta}-\frac{\Sigma_r}{r}\right)^2+\frac{2\Sigma_\eta\Sigma_r}{\eta r}(1-R_{\eta r}),$$

or
$$\frac{2\Sigma_\eta\Sigma_r}{\eta r}(1-R_{\eta r})=\Sigma_\varpi^2-\left(\frac{\Sigma_\eta}{\eta}-\frac{\Sigma_r}{r}\right)^2 \quad \text{.....(xxiv).}$$

This equation gives the numerical value of $R_{\eta r}$ as soon as r, η, Σ_r, Σ_η, Σ_ϖ have been determined, no matter in what way.

Assuming the simple formulae for Σ_r, Σ_η and the formula (xxiii) for Σ_ϖ, we get a corresponding formula

$$2\eta r(1-\eta^2)(1-r^2)(1-R_{\eta r})=\eta^2-r^2-(\eta-r)^2(1+\eta r)^2 \text{.....(xxv).}$$

* *Phil. Trans.* A, Vol. 191, p. 245. † *Loc. cit.* p. 20.

Problem VI. To find the standard deviation of the values of ζ defined by the equation

$$\zeta = \eta^2 - r^2.$$

We have $\quad \frac{1}{2}\delta\zeta = \eta\delta\eta - r\delta r.$

Hence $\quad \frac{1}{4}\Sigma_\zeta{}^2 = \eta^2\Sigma_\eta{}^2 + r^2\Sigma_r{}^2 - 2\eta r\Sigma_\eta\Sigma_r R_{\eta r}.$

But $$\Sigma_\varpi = \frac{\Sigma_\eta{}^2}{\eta^2} + \frac{\Sigma_r{}^2}{r^2} - \frac{2\Sigma_\eta\Sigma_r R_{\eta r}}{\eta r},$$

$$\therefore\ \eta^2 r^2\Sigma_\varpi{}^2 = r^2\Sigma_\eta{}^2 + \eta^2\Sigma_r{}^2 - 2\eta r\Sigma_\eta\Sigma_r R_{\eta r}.$$

Hence $\quad \frac{1}{4}\Sigma_\zeta{}^2 = (\eta^2 - r^2)(\Sigma_\eta{}^2 - \Sigma_r{}^2) + \eta^2 r^2\Sigma_\varpi{}^2$ .....(xxvi).

As in equation (xxiv) this is a formula always applicable. The formula for Σ_ζ which corresponds to equation (xxiii) for Σ_ϖ is therefore

$$\frac{1}{4}\Sigma_\zeta{}^2 = \frac{\eta^2 - r^2}{N}\{(1-\eta^2)^2 - (1-r^2)^2 + 1\} \text{.....(xxvii).}$$

Problem VII. To find the standard deviation of the values of ϑ defined by the equation

$$\vartheta = \eta - r.$$

Proceeding exactly as before, we get

$$\Sigma_\vartheta{}^2 = \Sigma_\eta{}^2 + \Sigma_r{}^2 - 2\Sigma_\eta\Sigma_r R_{\eta r},$$

giving $\quad \eta r\Sigma_\vartheta{}^2 = (\eta - r)\{r\Sigma_\eta{}^2 - \eta\Sigma_r{}^2\} + \eta^2 r^2\Sigma_\varpi{}^2$ .....(xxviii),

applicable in all cases.

The corresponding simple formula is

$$\eta r\Sigma_\vartheta{}^2 = \frac{\eta - r}{N}\{r(1-\eta^2)^2 - \eta(1-r^2)^2 + \eta + r\} \text{ .....(xxix).}$$

Statistical Illustrations.

I proceed to discuss, by appeals to actual statistical experience, the degree of closeness to which the simple formulae (xxiii), (xxvii) and (xxix), may be used instead of the complete formulae (xxii), (xxvi), and (xxviii).

If, as regards any frequency distribution, we have tabulated for each x-array of y's the n_{x_ρ}, y_{x_ρ} and $\sigma_{n_{x_\rho}}$, we may proceed to calculate Σ_ϖ from the formula (xxii) without further reference to the sub-groups of the correlation table. This follows at once since we have, in general,

$$\begin{aligned} Np_{m1} &= S\{n_{x_\rho y_s}(x_\rho - \bar{x})^m (y_s - \bar{y})\} \\ &= S\{n_{x_\rho y_s}(x_\rho - \bar{x})^m (y_s - y_{x_\rho})\} + S\{n_{x_\rho y_s}(x_\rho - \bar{x})^m (y_{x_\rho} - \bar{y})\} \\ &= N\lambda_{m1}, \end{aligned}$$

[since $S\{n_{x_\rho y_s}(y_s - y_{x_\rho})\} = 0$], hence

$$p_{m,1} = \lambda_{m1} \text{ .....(xxx).}$$

Also we have

$$\begin{aligned} Np_{m2} &= S\{n_{x_\rho y_s}(x_\rho-\bar{x})^m(y_s-\bar{y})^2\} \\ &= S\{n_{x_\rho y_s}(x_\rho-\bar{x})^m(y_s-y_{x_\rho}+y_{x_\rho}-\bar{y})^2\} \\ &= S\{n_{x_\rho y_s}(x_\rho-\bar{x})^2(y_s-y_{x_\rho})^2\}+S\{n_{x_\rho y_s}(x_\rho-\bar{x})^m(y_{x_\rho}-\bar{y})^2\} \\ &\qquad +2S\{n_{x_\rho y_s}(x_\rho-\bar{x})^m(y_{x_\rho}-\bar{y})(y_s-y_{x_\rho})\} \\ &= S\{n_{x_\rho}\sigma_{n_{x_\rho}}{}^2(x_\rho-\bar{x})^m\}+N\lambda_{m2}, \end{aligned}$$

hence

$$p_{m2}=\lambda_{m2}+\frac{1}{N}S\{n_{x_\rho}\sigma_{n_{x_\rho}}{}^2(x_\rho-\bar{x})^m\} \quad \text{.....(xxxi).}$$

Thus the formula for Σ_{ϖ} contains only λ's and χ_1, χ_3 which, by their definition, are calculable when n_{x_ρ}, y_{x_ρ}, $\sigma_{n_{x_\rho}}$ are known for every array, together with product moments of the form p_{m1}, p_{m2} which are also calculable if we use equations (xxx) and (xxxi).

In his memoir Karl Pearson discusses four actual frequency distributions which exhibit skew correlation and which he names illustrations *A*, *B*, *C*, *D*. For these distributions he has given the numerical values of many of the statistical constants and has also tabled the n_{x_ρ}, y_{x_ρ}, $\sigma_{n_{x_\rho}}$, for each array, to four decimal places in the cases of illustrations *A*, *B*, *C*, and to three decimal places in the case of illustration *D*. It is thus relatively easy to calculate Σ_{ϖ} from the formula (xxii) for these four cases. Starting with the tabled values of n_{x_ρ}, y_{x_ρ}, $\sigma_{n_{x_\rho}}$, I recalculated all the constants necessary, keeping six decimal places throughout the arithmetical processes and using Sheppard's corrections or not, according as Karl Pearson did or did not use them in his work. I have tabled all the constants to four decimal places, but, whenever any constant was used to obtain a new quantity, its value as obtained by me to six decimal places was used.

Although Karl Pearson obtained his constants by a different arithmetical process, and his results are necessarily more accurate, since he has kept six decimal places throughout, yet, wherever the values of constants given by my work may be compared with his, they agree to at least four decimal places excepting in the case of illustration *C*, in which the general agreement is only as far as the third decimal figure. This illustration has given throughout more irregular results than the others, due probably to the disturbing influence of the ecdyses.

Having obtained the necessary constants I proceeded to calculate Σ_{ϖ} for each case from the formula (xxii).

It remained to test the closeness to which the simple formulae (xxvii), (xxix) might be used instead of the formulae (xxvi), (xxviii) for Σ_ζ, Σ_λ. For this purpose Karl Pearson's four illustrations are again extremely convenient since these are the only published frequency distributions for which Σ_η has been calculated from the complex formula. In each of the four cases I substituted in the formulae (xxvi) and (xxviii) for Σ_{ϖ}, Σ_η the values found from the complex formulae, and

for Σ_r the value found from the usual approximate formula $\Sigma_r^2 = \frac{(1-r^2)^2}{N}$. Thus values for Σ_ϖ, Σ_ζ, Σ_ϑ were found which are certainly true values excepting in so far as the formula for Σ_r^2 is only approximate.

Writing E_ϖ, E_ζ, E_ϑ for the probable errors thence deduced we have $E_\varpi = \cdot 67449\Sigma_\varpi$, $E_\zeta = \cdot 67449\Sigma_\zeta$, $E_\vartheta = \cdot 67449\Sigma_\vartheta$.

The standard deviations and probable errors were now found from the simple formulae (xxiii), (xxvii) and (xxix) and the resulting values denoted by Σ'_ϖ, Σ'_ζ, Σ'_ϑ, E'_ϖ, E'_ζ, E'_ϑ. Finally the ratios $\frac{\varpi}{E'_\varpi}$, $\frac{\zeta}{E'_\zeta}$, $\frac{\vartheta}{E'_\vartheta}$ were calculated and the significance or non-significance of the deviations from linearity judged by the values of these ratios.

Exhibiting the values of the constants thus deduced; we have

Illustration A. On the Skew Correlation between Number of Branches to the Whorl and Position of the Whorl on the Spray in the case of *Asperula odorata.*

The position of the whorl is the x-character and the number of branches to the whorl is the y-character. Taking the values of n_{x_p}, y_{x_p}, $\sigma_{n_{x_p}}$ as given by Pearson, we have

	n_{x_p}	y_{x_p}	$\sigma_{n_{x_p}}$
x_1	150	6·7800	·8553
x_2	150	6·8133	·8437
x_3	150	6·8133	·9047
x_4	142	6·4859	·8780
x_5	87	6·1724	·8605

We find, the quantities being determined without the use of Sheppard's corrections,

$$N = 679,$$

$$\bar{x} = 2\cdot 8027, \qquad \bar{y} = 6\cdot 6554,$$
$$\sigma_x = 1\cdot 3369, \qquad \sigma_y = \cdot 8978,$$
$$\nu_2 = 1\cdot 7873, \qquad \mu_2 = \cdot 8061.$$
$$\nu_4 = 5\cdot 8417,$$

$$\sigma_M^2 = \lambda_2 = \cdot 0503, \qquad p_{11} = \cdot 2492,$$
$$\lambda_4 = \cdot 0075, \qquad p_{21} = \cdot 8964,$$
$$\lambda_{12} = \cdot 0342, \qquad p_{22} = 1\cdot 4906.$$
$$\lambda_{22} = \cdot 1678,$$

$$\sigma_M = \cdot 2244, \qquad \eta = \cdot 2499, \qquad r = \cdot 2076,$$
$$\chi_1 = \cdot 9909, \qquad \chi_3 = \cdot 9771.$$

Substituting these values in formula (xxii) we may exhibit the result so as to show the numerical contribution of each term in the form

$$\Sigma_{\varpi}^2 = \frac{1}{N}\{7{\cdot}1970 - {\cdot}2928 + {\cdot}9870 - 1{\cdot}1980 - {\cdot}0129 + {\cdot}2704 - {\cdot}5673 + {\cdot}5492\}$$

$$= \frac{1}{N}\{7{\cdot}1970 - {\cdot}5038 - {\cdot}3099 + {\cdot}5492\}$$

$$= \frac{1}{N}\{7{\cdot}1970 - {\cdot}2544\} = \frac{1}{N}\{6{\cdot}9426\} \quad \text{.....(xxxii).}$$

This gives $\Sigma_{\varpi} = {\cdot}1011, \qquad E_{\varpi} = {\cdot}0682.$

Pearson gives $E_{\eta} = {\cdot}0242.$

Hence $\Sigma_{\eta} = {\cdot}0359.$

From the formula $\Sigma_r^2 = \dfrac{(1-r^2)^2}{N},$

$$\Sigma_r^2 = {\cdot}0013.$$

Hence from formulae (xxvi) and (xxviii) we get

$$\Sigma_{\zeta} = {\cdot}0108, \qquad \Sigma_{\mathfrak{Z}} = {\cdot}0217.$$

Hence, finally $E_{\varpi} = {\cdot}0682, \qquad E_{\zeta} = {\cdot}0073, \qquad E_{\mathfrak{Z}} = {\cdot}0147,$

while, from the simple formulae (xxiii), (xxvii), (xxix) we get

$$E'_{\varpi} = {\cdot}0694, \qquad E'_{\zeta} = {\cdot}0071, \qquad E'_{\mathfrak{Z}} = {\cdot}0150.$$

These give $\dfrac{\varpi}{E'_{\varpi}} = 2{\cdot}6728, \qquad \dfrac{\zeta}{E'_{\zeta}} = 2{\cdot}7389, \qquad \dfrac{\mathfrak{Z}}{E'_{\mathfrak{Z}}} = 2{\cdot}8236.$

Illustration B. On the Correlation between Age and Head Height in Girls.

Age is taken as the x-character and height of head as the y-character.

We have in this case

x_ρ	n_{x_ρ}	x_{x_ρ}	$\sigma_{x_{x_\rho}}$
3— 4	1	115·2500	0·0000
4— 5	7	116·9643	5·7706
5— 6	18	117·4722	5·8552
6— 7	40	119·1000	5·9282
7— 8	76	120·3026	5·9764
8— 9	125	121·6340	5·2732
9—10	177	121·7246	6·7754
10—11	235	122·8160	5·9306
11—12	261	123·1427	6·4178
12—13	309	123·8908	6·4122
13—14	263	124·8622	6·7178
14—15	198	125·7146	7·1730
15—16	214	126·1565	6·9326
16—17	162	126·5340	7·7392
17—18	95	126·9132	6·3358
18—19	61	127·0205	6·2470
19—20	13	129·5577	9·6812
20—21	7	123·8214	5·0622
21—22	8	126·5000	8·2828
22—23	2	125·2500	1·9148

We find, using Sheppard's corrections throughout,

$$N = 2272,$$

$$\begin{aligned} \bar{x} &= 12{\cdot}7007, & \bar{y} &= 124{\cdot}0467, \\ \sigma_x &= 3{\cdot}0648, & \sigma_y &= 6{\cdot}9083, \\ \nu_2 &= 9{\cdot}3931, & \mu_2 &= 47{\cdot}7239. \\ \nu_4 &= 239{\cdot}1571, \end{aligned}$$

$$\begin{aligned} \sigma_M^2 = \lambda_2 &= 4{\cdot}3822, & p_{11} &= 6{\cdot}2274, \\ \lambda_4 &= 62{\cdot}3991, & p_{31} &= 148{\cdot}8952, \\ \lambda_{13} &= 80{\cdot}4155, & p_{22} &= 510{\cdot}1883. \\ \lambda_{22} &= 107{\cdot}3209, \end{aligned}$$

$$\sigma_M = 2{\cdot}0934, \qquad \eta = {\cdot}3030, \qquad r = {\cdot}2941,$$

$$\chi_1 = {\cdot}9993, \qquad \chi_3 = {\cdot}9953.$$

Hence we deduce from formula (xxii)

$$\begin{aligned} \Sigma_\varpi^2 &= \frac{1}{N}\{{\cdot}6688 - {\cdot}0724 + {\cdot}4546 - {\cdot}4035 \\ &\qquad + {\cdot}0623 + {\cdot}0533 - {\cdot}1964 \\ &\qquad + {\cdot}0867\} \\ &= \frac{1}{N}\{{\cdot}6688 - {\cdot}0213 - {\cdot}0808 + {\cdot}0867\} \\ &= \frac{1}{N}\{{\cdot}6688 - {\cdot}0154\} = \frac{1}{N}\{{\cdot}6534\} \quad \text{.....(xxxiii).} \end{aligned}$$

This gives $\Sigma_\varpi = {\cdot}0170, \qquad E_\varpi = {\cdot}0114.$

Pearson gives $E_\eta = {\cdot}0129.$

Hence $\Sigma_\eta = {\cdot}0191.$

From the formula $\Sigma_r^2 = \dfrac{(1-r^2)^2}{N},$

$$\Sigma_r^2 = {\cdot}0004.$$

Hence from formulae (xxvi) and (xxviii) we get

$$\Sigma_\zeta = {\cdot}0030, \qquad \Sigma_\vartheta = {\cdot}0051.$$

Hence, finally

$$E_\varpi = {\cdot}0114, \qquad E_\zeta = {\cdot}0020, \qquad E_\vartheta = {\cdot}0034,$$

while, from the simple formulae (xxiii), (xxvii) and (xxix) we get

$$E'_\varpi = {\cdot}0116, \qquad E'_\zeta = {\cdot}0021, \qquad E'_\vartheta = {\cdot}0034.$$

These give

$$\frac{\varpi}{E'_\varpi} = 2{\cdot}5752 \qquad \frac{\zeta}{E'_\zeta} = 2{\cdot}5879, \qquad \frac{\vartheta}{E'_\vartheta} = 2{\cdot}5973.$$

Illustration C. On the Skew Correlation between Size of Cell and Size of Body in *Daphnia magna.*

Length of body is taken for the x-character and size of cell for the y-character. We have

x_p	n_{x_p}	y_{x_p}	$\sigma_{n_{x_p}}$
1	20	5·3000	0·9092
2	150	5·8333	1·6620
3	100	7·7900	1·6860
4	80	8·0500	2·1539
5	150	9·4733	2·6694
6	140	8·4357	1·7087
7	230	8·5957	1·9384
8	120	10·2667	1·9439
9	180	10·7611	2·4469
10	150	11·0267	2·6949
11	150	10·9533	2·4470
12	140	9·1000	1·7947
13	120	9·0000	1·8619
14	220	10·0364	1·8894
15	60	10·3167	1·5165

We find, using Sheppard's corrections for the cell sizes, but not for the body lengths,

$$N = 2010,$$

$$\bar{x} = 8{\cdot}5025, \qquad \bar{y} = 9{\cdot}2687,$$
$$\sigma_x = 3{\cdot}8648, \qquad \sigma_y = 2{\cdot}5417,$$
$$\nu_2 = 14{\cdot}9366, \qquad \mu_2 = 6{\cdot}4604.$$
$$\nu_4 = 432{\cdot}7695,$$
$$\sigma_M^2 = \lambda_2 = 2{\cdot}1136, \qquad p_{11} = 3{\cdot}8929,$$
$$\lambda_4 = 15{\cdot}1375, \qquad p_{31} = 127{\cdot}3503,$$
$$\lambda_{13} = 27{\cdot}8071, \qquad p_{22} = 108{\cdot}1612.$$
$$\lambda_{22} = 56{\cdot}3594,$$

$$\sigma_M = 1{\cdot}4538, \qquad \eta = {\cdot}5720, \qquad r = {\cdot}3963,$$
$$\chi_1 = {\cdot}8918, \qquad \chi_3 = {\cdot}7574.$$

Substituting in formula (xxii) we get

$$\Sigma_w^2 = \frac{1}{N}\{3{\cdot}3108 - {\cdot}2651 + {\cdot}8098 - 1{\cdot}2303 + {\cdot}0971 - {\cdot}3796 - {\cdot}6074 + {\cdot}7752\}$$

$$= \frac{1}{N}\{3{\cdot}3108 - {\cdot}6855 - {\cdot}8898 + {\cdot}7752\}$$

$$= \frac{1}{N}\{3{\cdot}3108 - {\cdot}8001\} = \frac{1}{N}\{2{\cdot}5106\} \quad \text{.....(xxxiv).}$$

This gives $\Sigma_w = {\cdot}0353, \qquad E_w = {\cdot}0238.$

Pearson gives $E_\eta = {\cdot}0097.$

Hence $\Sigma_\eta = {\cdot}0144.$

From the formula $$\Sigma_r^2 = \frac{(1-r^2)^2}{N},$$

$$\Sigma_r^2 = \cdot 0004.$$

Hence from formulae (xxvi) and (xxviii) we get

$$\Sigma_\zeta = \cdot 0125, \qquad \Sigma_\eth = \cdot 0138.$$

Hence, finally

$$E_\varpi = \cdot 0238, \qquad E_\zeta = \cdot 0084, \qquad E_\eth = \cdot 0093,$$

while, from the simple formulae (xxiii), (xxvii), (xxix) we get

$$E'_\varpi = \cdot 0274, \qquad E'_\zeta = \cdot 0107, \qquad E'_\eth = \cdot 0114.$$

These give

$$\frac{\varpi}{E'_\varpi} = 13 \cdot 4052, \qquad \frac{\zeta}{E'_\zeta} = 15 \cdot 8934, \qquad \frac{\eth}{E'_\eth} = 15 \cdot 3972.$$

Illustration D. On the Skew Correlation between Number of Branches to the Whorl and Position of the Whorl on the stem in *Equisetum arvense.*

Position of whorl is taken as the x-character and number of branches to the whorl as y-character. We have

x_ρ	n_{x_ρ}	y_{x_ρ}	$\sigma_{y_{x_\rho}}$
1	126	7·619	2·360
2	126	9·294	1·273
3	126	9·627	1·187
4	126	9·730	1·151
5	126	9·643	1·158
6	124	9·427	1·375
7	123	8·732	1·781
8	121	7·297	2·291
9	119	5·555	2·553
10	110	3·964	2·199
11	97	2·443	1·506
12	67	1·866	0·960
13	39	1·462	0·746
14	12	1·333	0·471
15	4	1·250	0·433
16	2	1·000	0·000

Not using Sheppard's corrections for either character, we find

$$N = 1448,$$

$$\begin{aligned} \bar{x} &= 6 \cdot 4033, & y &= 7 \cdot 2169, \\ \sigma_x &= 3 \cdot 5426, & \sigma_y &= 3 \cdot 2785, \\ \nu_2 &= 12 \cdot 5500, & \mu_2 &= 10 \cdot 7486. \\ \nu_4 &= 319 \cdot 5158, & & \\ \sigma_M^2 = \lambda_2 &= 7 \cdot 7838, & p_{11} &= 8 \cdot 2256, \\ \lambda_4 &= 140 \cdot 4409, & p_{31} &= 205 \cdot 0840, \\ \lambda_{13} &= 152 \cdot 2416, & p_{22} &= 205 \cdot 2746. \\ \lambda_{22} &= 171 \cdot 1280, & & \end{aligned}$$

$$\sigma_M = 2 \cdot 7899, \qquad \eta = \cdot 8510, \qquad r = \cdot 7082,$$

$$\chi_1 = \cdot 6437, \qquad \chi_2 = \cdot 7992.$$

Hence from formula (xxii) we have

$$\Sigma_{\varpi}^2 = \frac{1}{N}\{\cdot6128 - \cdot2428 + 1\cdot0134 - \cdot9598 - \cdot1705 + \cdot9708 - \cdot6241 + \cdot0173\}$$

$$= \frac{1}{N}\{\cdot6128 - \cdot1893 + \cdot1762 + \cdot0173\}$$

$$= \frac{1}{N}\{\cdot6128 + \cdot0042\} = \frac{1}{N}\{\cdot6170\} \quad \text{.....(xxxv).}$$

This gives $\Sigma_{\varpi} = \cdot0206, \qquad E_{\varpi} = \cdot0139.$

Pearson gives $E_{\eta} = \cdot0054.$

Hence $\Sigma_{\eta} = \cdot0080.$

From the formula $\Sigma_r^2 = \dfrac{(1-\gamma^2)^2}{N},$

$$\Sigma_r^2 = \cdot0002.$$

Hence from formulae (xxvi) and (xxviii) we get

$$\Sigma_{\zeta} = \cdot0229, \qquad \Sigma_{\vartheta} = \cdot0153.$$

Hence, finally

$$E_{\varpi} = \cdot0139, \qquad E_{\zeta} = \cdot0154, \qquad E_{\vartheta} = \cdot0103,$$

while, from the simple formulae (xxiii), (xxvii) and (xxix) we get

$$E'_{\varpi} = \cdot0139, \qquad E'_{\zeta} = \cdot0152, \qquad E'_{\vartheta} = \cdot0102.$$

These give

$$\frac{\varpi}{E'_{\varpi}} = 13\cdot2341, \qquad \frac{\zeta}{E'_{\zeta}} = 14\cdot6291, \qquad \frac{\vartheta}{E'_{\vartheta}} = 13\cdot9785.$$

In order to appreciate the significance of the results in these four cases they are collected and tabulated as follows:

$$\left[\text{Note: } \varpi = \log_e \frac{\eta}{r} = \log_{10} \frac{\eta}{r} \times \log_e 10\right]$$

	ϖ	ζ	ϑ	$\frac{\varpi}{E'_{\varpi}}$	$\frac{\zeta}{E'_{\zeta}}$	$\frac{\vartheta}{E'_{\vartheta}}$		E_{ϖ}	E_{ζ}	E_{ϑ}
A	·1856	·0194	·0423	2·6728	2·7389	2·8236	E'	·0694	·0071	·0150
							E	·0682	·0073	·0147
							$E'\sim E$	·0012	·0002	·0003
B	·0298	·0053	·0089	2·5752	2·5879	2·5973	E'	·0116	·0021	·0034
							E	·0114	·0020	·0034
							$E'\sim E$	·0002	·0001	·0000
C	·3669	·1701	·1757	13·4052	15·8934	15·3972	E'	·0238	·0107	·0114
							E	·0274	·0084	·0093
							$E'\sim E$	·0036	·0023	·0021
D	·1836	·2226	·1438	13·2341	14·6291	13·9785	E'	·0139	·0152	·0102
							E	·0139	·0154	·0103
							$E'\sim E$	·0000	·0002	·0001

We conclude from this table:

I. As regards the linearity of the regression in the statistical examples chosen, illustrations C and D are certainly skew. The skewness of these examples is obvious either from a glance at the diagrams accompanying Karl Pearson's memoir or from his discussion of the cubic and quartic regression curves for these cases. Illustrations A and B give $\frac{\varpi}{E_{\varpi}}$, $\frac{\zeta}{E_{\zeta}}$, $\frac{\vartheta}{E_{\vartheta}}$ values between 2 and 3. The limiting value of these ratios which should denote significance is a matter which can only be decided as a result of large statistical experience. When the ratio gets as low as 2 we are getting on the border line and for cases A and B we can only say that the skewness is probably significant. This is the kind of result we should expect from the corresponding diagrams of Karl Pearson's memoir. He finds, however, that he obtains so much better fits by curves of higher order that he concludes that the regression of A is parabolic and of B cubic.

II. If we refer to equations (xxxii), (xxxiii), (xxxiv), (xxxv) we see exhibited the numerical contribution of each term of formula (xxii) for $\Sigma_{\varpi}{}^2$. As regards the numerical values of the quantities which vanish for normal correlation we see that they by no means become zero in the examples chosen.

Also in these equations I have grouped together the three p terms and the three λ terms, but the numerical values of the separate terms are still considerable. When the whole expression for $\Sigma_{\varpi}{}^2$ is summed however we find that the last three terms of these numerical equations so nearly cancel that the difference between E_{ϖ} and E'_{ϖ} is at the most ·004, a quantity which is of no significance in the value of a probable error. This is exactly the kind of result Karl Pearson found in comparing the values of E_{η} as found from the complex formula and the simple formula. There is thus exactly the same justification for the use of formula (xxiii) as for the formula $\Sigma_{\eta}{}^2 = \frac{(1-\eta^2)^2}{N}$.

Comparing the values of E_{ζ}, E_{ϑ} with E'_{ζ}, E'_{ϑ} we see that the agreement is closer still, the difference being at the most of order ·002; and if we except illustration C the agreement in the values of E_{ζ}, E_{ϑ} is remarkably close, being of the order ·0003 at the most.

Thus our numerical work leads us to the conclusion that we have the same justification for the use of the simple formulae (xxiii), (xxvii), and (xxix) for E_{ϖ}, E_{ζ}, E_{ϑ} as for the use of the accepted simple formulae for E_r, E_{η}.

These simple formulae have been obtained by making approximations of a statistical nature, i.e. approximations suggested entirely by statistical experience apart from arithmetic. I proceed to examine the formulae thus obtained to see if any simplification can be made by arithmetical approximation.

Accepting the simple formula (xxiii) for Σ_ϖ, we get

$$\frac{\varpi^2}{N\Sigma_\varpi{}^2} = \frac{\left[\log_e \frac{\eta}{r}\right]^2}{\frac{1}{r^2} - \frac{1}{\eta^2}} = \frac{\left[\log_e \left(1 + \frac{\vartheta}{r}\right)\right]^2}{\frac{1}{r^2} - \frac{1}{(r+\vartheta)^2}}.$$

Assuming $\frac{\vartheta}{r} < 1$, we can expand this expression in a series of ascending powers of ϑ, and, neglecting cubes and higher powers, we get

$$\frac{\varpi^2}{N\Sigma_\varpi{}^2} = \frac{r\vartheta}{2} + \frac{\vartheta^2}{4} \quad \text{..... (xxxvi).}$$

Proceeding in exactly the same way from equations (xxvii) and (xxix), we get

$$\frac{\zeta^2}{N\Sigma_\zeta{}^2} = \frac{r\vartheta}{2} + \frac{\vartheta^2}{4} + \frac{\vartheta^2}{4} \cdot 8r^2(1-r^2) \text{..... (xxxvii),}$$

$$\frac{\vartheta^2}{N\Sigma_\vartheta{}^2} = \frac{r\vartheta}{2} + \frac{\vartheta^2}{4} + \frac{\vartheta^2}{4} \cdot (1+3r^2)(1-r^2) \text{ ..... (xxxviii).}$$

When $\frac{\vartheta}{r}$ is small these approximate formulae will be very nearly accurate, and they exhibit the arithmetical reason why $\frac{\varpi}{E'_\varpi}$, $\frac{\zeta}{E'_\zeta}$, $\frac{\vartheta}{E'_\vartheta}$ work out nearly equal and, moreover, we see in what manner they will differ when they are not equal. In our statistical examples $\frac{\vartheta}{r}$ is small and we should expect from these second order approximations that $\frac{\zeta}{E'_\zeta}$, $\frac{\vartheta}{E'_\vartheta}$ would be greater than $\frac{\varpi}{E'_\varpi}$, which will be seen to be the case on referring to the table of values. Again, $\frac{\vartheta}{E'_\vartheta} \gtreqless \frac{\zeta}{E'_\zeta}$ according as $1 + 3r^2 \gtreqless 8r^2$, i.e. as $1 \gtreqless 5r^2$; if r^2 is nearly $\frac{1}{5}$, $\frac{\vartheta}{E'_\vartheta}$, $\frac{\zeta}{E'_\zeta}$ will be nearly equal, but we cannot say which will be the greater as the third order terms are then likely to become important in determining the difference. These conclusions are seen to be borne out by the table of values, and other peculiarities in the values might be explained in the same way.

When $\frac{\vartheta}{r}$ is small, equations (xxxvi), (xxxvii), and (xxxviii), obviously suggest the arithmetical approximation

$$\frac{\varpi}{E_\varpi} = \frac{\zeta}{E_\zeta} = \frac{\vartheta}{E_\vartheta} = \frac{\sqrt{N}}{\cdot 67449} \cdot \tfrac{1}{2}\sqrt{\zeta} \text{.....(xxxix)}$$

since $$\frac{r\vartheta}{2} + \frac{\vartheta^2}{4} = \frac{\eta - r}{4}\{2r + \eta - r\} = \frac{\eta^2 - r^2}{4}.$$

I append the values given by (xxxix), for the four statistical illustrations:

	A	B	C	D
$\frac{\vartheta}{r}$	·2039	·0302	·4433	·2016
$\frac{\sqrt{N}}{\cdot 67449} \cdot \frac{1}{2}\sqrt{\zeta}$	2·6883	2·5711	13·7078	13·3086

Even when $\frac{\vartheta}{r}$ is not very small

$$\frac{\zeta}{E'_\zeta} = \frac{\sqrt{N}}{\cdot 67449} \cdot \frac{1}{2}\sqrt{\zeta} \cdot \frac{1}{\sqrt{1 - 2(\eta^2 - r^2) + \eta^4 - r^4}},$$

and it is obvious that if there be any doubt as to linearity of regression, when $\frac{\vartheta}{r}$ is not small, then r and η must be both small and $\frac{\zeta}{E'_\zeta} = \frac{\sqrt{N}}{\cdot 67449} \cdot \frac{1}{2}\sqrt{\zeta}$ will still be a good arithmetical approximation for equation (xxvii).

General Conclusion.

Thus to test whether the regression of any frequency distribution is linear or not we have three possible methods, viz.:—to compare the value of either ϖ, ζ, or ϑ, with its probable error. As regards the best quantity to choose for this we may first say that ϖ has no advantages whatever. In favour of the use of ϑ we may say that the terms of ϑ are just η and r, which are the quantities in terms of which we naturally judge our distribution; we must be careful however in using formula (xxix) for Σ_ϑ to remember that r is supposed positive; i.e. if r is negative for any case we must change its sign before substituting in the formula and we then get the probable error of the difference between η and the absolute magnitude of r.

ζ however is the term occurring naturally in the work and is the one quantity of the three with a direct physical meaning; i.e. ζ gives the mean square deviation of the distribution from the regression line. This will probably be sufficient to assure that tests for linearity will be conducted in terms of ζ.

Finally,

(i) A simple test for linearity of regression which will be sufficient in very many cases is

$$\frac{\sqrt{N}}{\cdot 67449} \cdot \frac{1}{2}\sqrt{\zeta} < 2{\cdot}5.$$

(ii) For more exact work $\frac{\zeta}{E_\zeta}$, $\frac{\mathfrak{H}}{E_\mathfrak{H}}$ should be calculated from the formulae

$$\frac{\zeta}{E_\zeta} = \frac{\sqrt{N}}{\cdot 67449} \cdot \tfrac{1}{2}\sqrt{\zeta} \cdot \frac{1}{\sqrt{1+(1-\eta^2)^2-(1-r^2)^2}},$$

$$\frac{\mathfrak{H}}{E_\mathfrak{H}} = \frac{\sqrt{N}}{\cdot 67449} \cdot \tfrac{1}{2}\sqrt{\zeta} \cdot \frac{\sqrt{4\eta r}}{\eta + r} \cdot \frac{1}{\sqrt{1+\dfrac{r(1-\eta^2)^2-\eta(1-r^2)^2}{\eta+r}}}.$$

Each of these results gives an approximate formula for the same physical quantity (i.e. the number to be looked up in the table of the probability integral if we want the probability in favour of the distribution being linear).

If these ratios work out sufficiently nearly equal this constitutes some justification for the statistical approximations made in obtaining the formulae, but if on comparing the values of the ratios the agreement is unsatisfactory, and no mistake in the arithmetic can be detected, then the probable errors of E_ζ, $E_\mathfrak{H}$ must be calculated from the complete formulae (xxii), (xxvi), (xxviii) in terms of the subsidiary quantity ϖ and substituting for $\Sigma_{,\eta}{}^2$ its value as determined by the complete formula given by Karl Pearson.

THE LONG BARROW AND ROUND BARROW SKULLS IN THE COLLECTION OF THE DEPARTMENT OF COMPARATIVE ANATOMY, THE MUSEUM, OXFORD.

By E. H. J. SCHUSTER, M.A., New College, Oxford.

THE following work was undertaken at the desire of Professor Weldon. It is the first part of a descriptive catalogue of the British skulls in the collection of the Department of Comparative Anatomy, Oxford, and is an attempt to lay before the public in an accessible and partially digested form the material in this collection. Owing to the small number of the measurements of each group which it has been possible to take, the statistical constants deduced from them are not of very much value as they stand, but it is hoped that similar catalogues of other collections may be published which will enable one to deal with larger groups.

Professor Weldon and Professor Karl Pearson have written a descriptive catalogue of the skulls, which has been based to a certain extent on the manuscript catalogue prepared some years ago for the Department of Comparative Anatomy by Mr Hatchett Jackson. The account of the anatomical peculiarities was most kindly provided by Professor Thane.

Professor Thane's determinations of the sex have also been adhered to throughout.

The following measurements were taken to such extent as was possible in each individual case.

A. CRANIAL.

L'. Horizontal length. This was measured with instruments described in Miss Fawcett's memoir (*Biometrika*, Vol. I. p. 408).

L. Greatest length measured from the *glabella* to *occiput.*

L''. Greatest length measured from the *nasion.*

F. Ophryo-occipital length. Flower. Greatest length measured from the middle point between the frontal prominences.

B. *Greatest breadth* measured outside the squamosals.

B″. *Auricular breadth* of Virchow—distance between the upper borders of the auditory canal.

B′. *Smallest frontal breadth.*

H′. *Height.* From the anterior border of the foramen magnum to the point on the surface of the skull, in a line vertical to the horizontal plane.

H. *Auxiliary height.* From the anterior border of the foramen magnum to the bregma, i.e. the juncture of the sagittal and coronary sutures.

OH. *Auricular height,* measured with the vertical scale and sliding rod of the craniophor (*Biometrika*, Vol. I. p. 414).

LB. *Basicranial length,* measured from the anterior border of the foramen magnum to the nasion.

fml. *Greatest length* of *foramen magnum.*

fmb. *Greatest breadth* of *foramen magnum.*

U. *Horizontal circumference,* measured with the steel band measure over the superciliary ridges in front and the most projecting part of the skull behind.

S. *Sagittal circumference,* measured from the nasofrontal suture to the posterior border of the foramen magnum over the sagittal suture.

Q. *Vertical transverse circumference,* measured from the upper border of the ear opening of one side to that of the other, in a plane vertical to the horizontal plane.

The foregoing measurements are described in the *Archiv für Anthropologie*, Band XV., Braunschweig, 1884.

B. Facial Measurements.

G′H. *Upper face height,* measured from the nasion to the middle of the central process of the upper jaw between the middle incisor teeth, (i.e. the alveolar point).

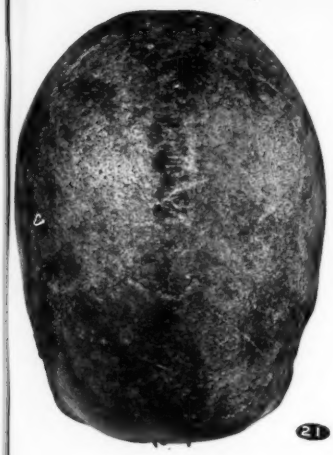
GB. *Face breadth,* being the distance from one zygomatic maxillary suture to the other. The measurement must be taken from the lower end of these sutures, from the lower front rim of one cheek-bone to that of the other.

J. *Bizygomatic breadth,* from the outermost point of one zygomatic arch to the like point on the other.

NH. *Nasal height,* measured from the nasion to the lowest edge of the pyriform aperture.

NB. *Nasal breadth,* greatest breadth of the nasal aperture, wherever it may be.

O_1. *Breadth of orbit* for both left (*L*) and right (*R*) eyes, measured from the "point lacrymal" on the inner side to the inner margin of the orbit on the other side, along a line parallel to the general slope of the orbit.



Long Barrow Crania.

N. verticalis.

123

135

138

185

186

207

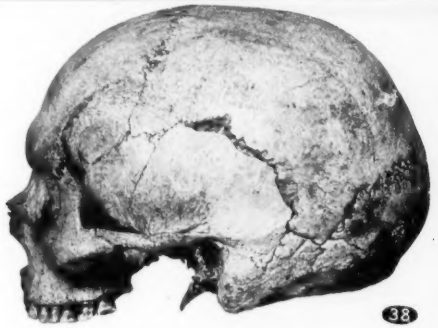
218

99

212

Round Barrow Crania.

N. verticalis.



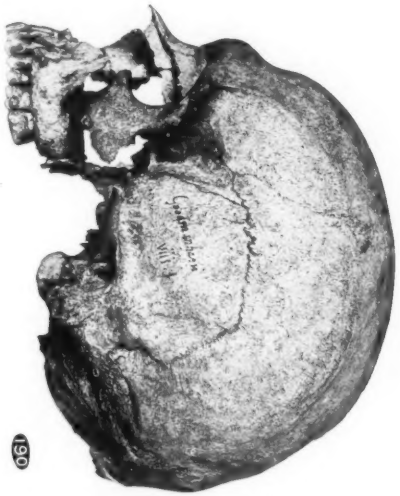
Long Barrow Crania.

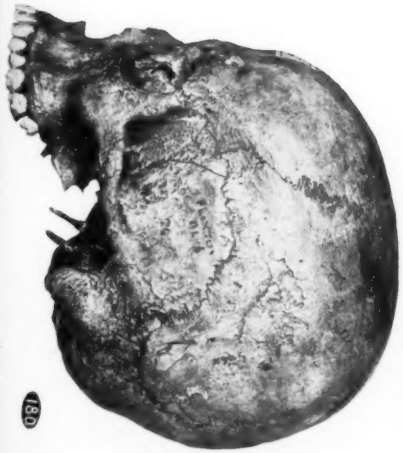
N. lateralis

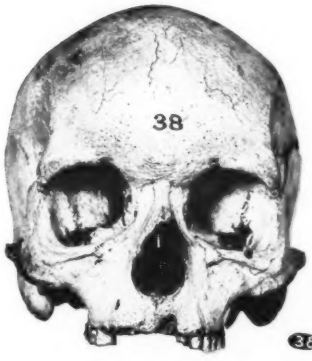
Round Barrow Crania.

N. lateralis.

Plate iv.

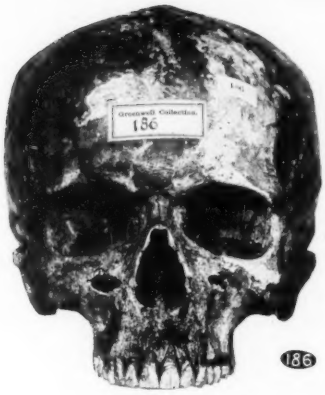


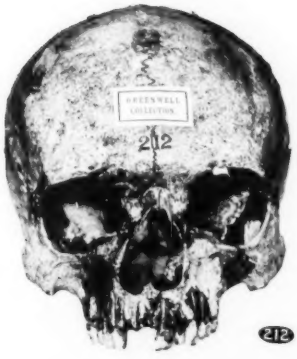




Round Barrow Crania.

N. facialis.





Round Barrow Crania.

N. facialis.

"Point lacrymal" is the point at which the hinder edge of the lacrymal groove meets the lacrymo-frontal suture.

O_2. *Greatest height of orbit*, for both (L) and (R) eyes, taken as closely as possible perpendicular to O_1.

G_1. *Length of palate*, measured from the point of the *Spina nasalis posterior* to the inner wall of the alveolar rim between the middle incisors.

G_2. *Breadth of palate* between the alveolar walls at the second molars.

GL. *Profile length* from the basion to the alveolar point.

W_1. *Condylar width.* Greatest width of mandibles at condyles, from the outside of one condyle to the outside of the other.

W_2. *Angle width.* Greatest width of mandibles at angles, from the outside of one angle to the outside of the other.

h_1. *Greatest height of mandible*, measured from the lowest median projection to the top of process between middle incisors.

f. *Distance between foramina mentalia.*

C. INDICES.

B/L'. *Cephalic index* for horizontal length.

B/L. *Cephalic index* for greatest length measured from the glabella.

H'/L'. *The ratio of height to length*, for horizontal length and the whole height.

H/L. *The ratio of height to length*, for the greatest length measured from the glabella and for the auxiliary height.

$G'H/GB$. The upper face height to face breadth index.

NB/NH. The nasal breadth to nasal height index.

O_2/O_1 (R), O_2/O_1 (L). The orbital height to orbital breadth index for both right and left eyes.

G_2/G_1. The palate breadth to palate length index.

P. The profile angle found with Ranke's Goniometer.

The description of the facial measurements, and of the indices calculated, is taken from Miss Fawcett's paper on the Naqada Crania (*Biometrika*, Vol. I.), and the lettering used in the tables of that memoir to denote these various measurements and indices has been retained. But it will be noticed that certain of the measurements have been omitted, and others slightly modified. Thus owing to the imperfect state of the teeth, in many cases, the face height was not measured. Also the orbital breadths have been measured from a different point on the inner side as it was found more convenient to do them in this way.

Owing to the few cases in which these determinations could have been made, it was not considered worth while to calculate the angles $A\angle$, $N\angle$, $B\angle$, θ_1, θ_2.

The measurement of the capacity was not attempted owing to the fragmentary nature of the majority of the skulls*.

* The following six were found fit for the measurement of the capacity which is given in cm.^{3}:
24, 1606. 26, 1496. 38, 1592. 83, 1667, 173, 1437. 180, 1502.

Table I. shews the means of the measurements thus taken, with the number of measurements in each case. Tables II. to IV. give the actual measurements. Plates I. to VI. give characteristic specimens of Long and Round Barrow Crania with a few abnormal cases.

TABLE I. *Showing means for each group of skulls and each measurement, with number of observations in each case.*

Measurement	Long Barrow				Round Barrow			
	Mean ♂	Number of observations ♂	Mean ♀	Number of observations ♀	Mean ♂	Number of observations ♂	Mean ♀	Number of observations ♀
L'	191·88	8	185·33	3	185·75	8	179·45	11
L	190·56	16	182·61	13	188·61	36	177·97	34
L''	188·44	16	181·50	12	185·03	32	175·88	30
F	187·15	17	183·96	12	187·01	37	178·63	35
B	142·44	18	138·62	12	144·77	31	136·56	34
B''	121·37	16	116·94	9	121·68	25	113·64	22
B'	98·94	16	94·09	11	98·27	32	95·98	29
H'	138·44	9	138·83	3	138	3	136·00	2
H	137·83	12	135·11	9	135·94	16	130·80	15
OH	120·67	9	118·00	3	123·62	8	115·91	11
LB	101·95	11	96·81	8	103·94	16	99·33	12
fml	35·68	11	34·50	6	36·97	15	35·44	9
fmb	27·73	11	30·25	6	31·00	15	28·79	7
U	534·87	16	518·71	7	537·85	27	506·43	30
S	384·77	13	382·00	8	376·53	15	365·30	15
Q	321·78	9	312·00	3	333·38	8	313·54	11
G'H	69·92	13	66·75	4	70·44	16	66·20	10
GB	95·88	12	92·75	4	97·42	12	90·29	7
J	134·00	3	132·5	1	133·50	3	—	—
NH	49·43	15	47·00	7	50·18	14	48·22	9
NB	24·07	15	22·79	7	24·61	14	23·50	9
O_1 *L*	38·12	8	36·87	4	39·03	15	38·78	9
O_1 *R*	38·28	9	37·87	4	40·00	21	39·35	10
O_2 *L*	31·81	8	30·62	4	32·46	14	32·67	9
O_2 *R*	31·28	9	30·87	4	31·79	21	32·19	8
GL	95·33	9	92·62	4	96·00	8	93·43	7
G_1	55·31	8	57·00	1	55·59	11	50·69	8
G_2	43·70	5	36·00	1	40·73	15	39·11	9
W_1	121·21	7	113·83	3	123·38	13	117·35	10
W_2	104·90	10	92·92	6	105·04	24	94·85	13
h_1	34·13	15	31·80	5	33·83	21	30·89	17
f	45·67	15	85·5	3	45·23	22	43·69	16
B/L'	74·38	8	74·33	3	77·88	8	76·36	11
B/L	74·93	16	76·33	12	76·70	30	77·73	33
H'/L'	72·25	8	74·66	3	73·67	3	74·50	2
H/L	72·73	11	74·00	8	72·81	16	74·85	13
G'H/GB	71·44	9	81·00	1	72·83	12	73·50	6
NB/NH	49·00	15	49·10	6	49·14	14	48·89	9
$O_2/O_1 : L$	83·37	8	81·66	3	82·87	15	84·22	9
$O_2/O_1 : R$	81·89	9	80·66	3	79·60	20	82·12	8
G_2/G_1	78·40	5	—	—	73·82	11	76·75	8
P	83·00°	5	—	—	84·14°	7	84·00°	6

MEAS

Number of Skull	*L'*	*L*	*L''*	*F*	*B*	*B''*	*B'*	*H'*	*H*	*OH*	*LB*	*fml*	*fmb*	*U*	*S*	*Q*	*G'H*	*GB*	*J*	*NH*
3	—	—	—	182·5	145	120·5	97·5	—	—	—	—	—	—	523	—	—	—	—	—	—
4	—	188	183	185	130·5	—	98·5	—	—	—	—	—	—	518	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	—	198	196·5	—	137·5	—	—	—	—	—	—	—	—	537	—	—	77	—	—	51
21	197	196	194	197	140	119·5	101·5	142	142·5	122	106·5	36	31	548	400	326	—	92	—	51
22	191	190	189	191·5	137	119	95	135	134	116	96·5	36·5	—	538	399	310	64	98	136	48·5
25	—	—	188	—	141	114·5	99	137	137·5	117	103	—	—	—	—	312	70	99·5	127	52·5
38	197	198	194	194	143	123	98	139	139	120	106	33·5	29·5	546	399	322	—	93	139	50·5
48	—	208	205	208	137	124	103	—	—	—	—	—	—	—	394	—	81	—	—	55
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	70	—	—	49
59	—	190·5	187	187·5	142	115	—	—	—	—	—	—	—	538	374	—	71	91·5	—	50·5
62	—	194·5	189·5	193·5	—	—	—	—	—	—	—	33·5	28·5	—	—	—	—	—	—	—
65	182	181·5	180·5	178·5	133	116·5	100	126	125·5	111	96·5	37·5	28·5	511	362	301	70	92	—	49·5
72	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
77	198	197·5	193	193·5	142	121·5	102·5	143	144	127	102·5	39	31·5	550	400	325	68·5	96·5	—	48
80	194	192·5	190	182	149	125·5	97·5	137	136·5	118	108·5	37	30	553	377	321	76	97	—	56·5
81	—	—	—	187	152	123	98·5	—	146	—	—	—	—	540	366	—	—	—	—	—
82	—	184	182·5	180·5	142	123·5	98	—	133	—	94	34	33·5	527	384	—	60·5	—	—	41
83	186	185	183	186	149	123	97	146	146	126	104·5	36	33	535	390	333	67	99·5	—	42
84	—	181	—	175	152	128	100	—	130	—	103	39·5	32·5	535	358	—	69·5	99	—	50·5
86	190	188	184·5	188·5	147	124	96	141	140	129	100·5	30	27	541	399	346	64·5	94·5	—	46
87	—	176·5	175·5	171·5	145	121·5	98	—	—	—	—	—	—	518	—	—	—	98	—	—

LONG H

13	—	172	171·5	171·5	139·5	—	101	—	—	—	—	—	—	502	—	—	—	—	—	—
24	187	188	187	187·5	138	119	92	141·5	140	118	100·5	34	31	530	389	311	—	92·5	132·5	49
26	181	182	180·5	186·5	136	113	89·5	142	140·5	121	98	33·5	30·5	508	387	314	—	—	—	45·5
27	—	176·5	175	182	132	—	—	—	139·5	—	96·5	—	—	—	382	—	64·5	—	—	46
28	—	—	—	—	—	—	90	—	141·5	—	98	36·5	29·5	—	—	—	65·5	92	—	50
30	—	181·5	173·5	182·5	136	113·5	85·5	—	—	—	—	—	—	—	—	—	—	—	—	—
31	—	188·5	187	185·5	131	115·5	87·5	—	—	—	—	—	—	517	377	—	—	—	—	—
39	188	187	186	188	139·5	115	93	133	131·5	115	99	34	29·5	524	384	311	—	98	—	44
43	—	186	184	—	138	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46	—	193	190·5	196	—	—	100·5	—	—	—	—	—	—	—	—	—	—	—	—	—
75	—	190	188·5	194·5	145·5	122	97·5	—	135·5	—	89·5	—	—	540	405	—	—	—	—	—
78	—	178	174·5	180·5	135	113	98	—	121	—	93	—	—	510	362	—	65·5	—	—	44·5
79	—	169	—	170·5	146	120·5	—	—	128·5	—	—	35	30·5	—	—	—	—	—	—	—
85	—	182·5	180	182·5	147	121	100·5	—	138	—	100	34	30·5	—	370	—	71·5	88·5	—	50

LONG BARROW. ♂.

MEASUREMENTS

	J	NH	NB	O_1 L	O_1 R	O_2 L	O_2 R	GL	G_1	G_2	W_1	W_2	h_1	f	B/L′	B/L	H′/L′	H/L	G′H/GB	NB
	—	—	—	—	—	—	—	—	—	—	—	—	33	43·5	—	—	—	—	—	
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	69	—	—	—	
	—	—	—	—	—	—	—	—	—	—	—	—	33	45	—	—	—	—	—	
	—	—	—	—	—	—	—	—	—	—	—	115·5	35	46·5	—	—	—	—	—	
	—	51	22·5	—	—	—	—	—	55	—	—	—	41	46·5	—	69	—	—	—	
	—	51	22	37·5	37·5	33	33·5	—	—	—	—	—	—	—	71	71	72	72	—	
	136	48·5	22	37	37·5	30·5	31	86	53	—	—	—	—	—	72	72	71	71	65	
5	127	52·5	25·5	40·5	40·5	32·5	31·5	96	—	—	—	—	—	—	—	—	—	—	70	
	139	50·5	22·5	39·5	39·5	35	34·5	—	—	—	—	—	—	—	73	72	71	70	—	
	—	55	24·5	—	—	—	—	—	—	—	123·5	—	38	42	—	—	—	—	—	
	—	49	23·5	—	35·5	—	33	—	56·5	41	—	—	—	—	—	—	—	—	—	
5	—	50·5	23	38·5	—	32·5	—	—	—	—	111	105	37·5	45	—	75	—	—	78	
	—	—	—	—	—	—	—	—	—	—	—	99·5	31	46	—	66	—	—	—	
	—	49·5	20·5	36·5	36	31	31·5	97	55·5	—	—	—	33	45	73	73	69	69	76	
	—	—	—	—	42	—	28	—	—	—	—	—	—	—	—	—	—	—	—	
5	—	48	26·5	—	—	—	—	98	58	44	125·5	112	30·5	46·5	72	72	72	73	71	
	—	56·5	27	—	—	—	—	106·5	—	—	—	104	36·5	51·5	77	78	71	71	78	
	—	—	—	—	—	—	—	—	—	—	—	97·5	32·5	43	—	80	—	77	—	
	—	41	24·5	37·5	37	29	29	89	51	43	126·5	105·5	31·5	45	—	77	—	72	—	
5	—	42	25·5	—	39	—	29·5	98	57·5	47	121	99·5	35	46	80	81	78	79	67	
	—	50·5	27	—	—	—	—	96	—	—	119·5	108	32	45·5	—	84	—	72	70	
5	—	46	24·5	38	—	31	—	91·5	56	43·5	121·5	102·5	32·5	48	77	78	74	74	68	
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	82	—	—	—	

LONG BARROW. ♀.

	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	81	—	—	—	
5	132·5	49	24·5	36·5	37·5	31	31	—	—	—	—	—	—	—	74	73	75	74	—	
	—	45·5	23	36·5	—	29	—	—	—	—	—	—	—	—	75	75	78	77	—	
	—	46	21	—	—	—	—	85·5	—	—	—	—	—	—	—	75	—	79	—	
	—	50	22·5	35·5	36·5	31	31	98	57	36	—	—	—	—	—	—	—	—	—	
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	75	—	—	—	
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	70	—	—	—	
	—	44	22	—	38	—	30	—	—	—	117	86·5	30	47·5	74	74	71	71	—	
	—	—	—	—	—	—	—	—	—	—	104·5	85·5	28	43	—	74	—	—	—	
	—	—	—	—	—	—	—	—	—	—	117	97	—	39	—	—	—	—	—	
	—	—	—	—	—	—	—	—	—	—	—	96·5	37·5	47·5	—	76	—	72	—	
	—	44·5	22·5	—	—	—	—	93·5	—	—	—	89	30	37	—	76	—	68	—	
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	86	—	76	—	
5	—	50	24	39	39·5	31·5	31·5	93·5	—	—	—	103	33·5	42·5	—	81	—	75	81	48

LONG BARROW. ♂.

MEASUREMENTS

H'	*H*	*OH*	*LB*	*fml*	*fmb*	*U*	*S*	*Q*	*G'H*	*GB*	*J*	*NH*	*NB*	O_1 *L*	O_1 *R*	O_2 *L*	O_2 *R*	*GL*	G_1	G_2
—	—	—	—	—	—	523	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	518	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	537	—	—	77	—	—	51	22·5	—	—	—	—	—	55	—
142	142·5	122	106·5	36	31	548	400	326	—	92	—	51	22	37·5	37·5	33	33·5	—	—	—
135	134	116	96·5	36·5	—	538	399	310	64	98	136	48·5	22	37	37·5	30·5	31	86	53	—
137	137·5	117	103	—	—	—	—	312	70	99·5	127	52·5	25·5	40·5	40·5	32·5	31·5	96	—	—
139	139	120	106	33·5	29·5	546	399	322	—	93	139	50·5	22·5	39·5	39·5	35	34·5	—	—	—
—	—	—	—	—	—	—	394	—	81	—	—	55	24·5	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	70	—	—	49	23·5	—	35·5	—	33	—	56·5	41
—	—	—	—	—	—	538	374	—	71	91·5	—	50·5	23	38·5	—	32·5	—	—	—	—
—	—	—	—	33·5	28·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
126	125·5	111	96·5	37·5	28·5	511	362	301	70	92	—	49·5	20·5	36·5	36	31	31·5	97	55·5	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	42	—	28	—	—	—
143	144	127	102·5	39	31·5	550	400	325	68·5	96·5	—	48	26·5	—	—	—	—	98	58	44
137	136·5	118	108·5	37	30	553	377	321	76	97	—	56·5	27	—	—	—	—	106·5	—	—
—	146	—	—	—	—	540	366	—	—	—	—	—	—	—	—	—	—	—	—	—
—	133	—	94	34	33·5	527	384	—	60·5	—	—	41	24·5	37·5	37	29	29	89	51	43
146	146	126	104·5	36	33	535	390	333	67	99·5	—	42	25·5	—	39	—	29·5	98	57·5	47
—	130	—	103	39·5	32·5	535	358	—	69·5	99	—	50·5	27	—	—	—	—	96	—	—
141	140	129	100·5	30	27	541	399	346	64·5	94·5	—	46	24·5	38	—	31	—	91·5	56	43·5
—	—	—	—	—	—	518	—	—	—	98	—	—	—	—	—	—	—	—	—	—

LONG BARROW. ♀.

—	—	—	—	—	—	502	—	—	—	—	—	—	—	—	—	—	—	—	—	—
141·5	140	118	100·5	34	31	530	389	311	—	92·5	132·5	49	24·5	36·5	37·5	31	31	—	—	—
142	140·5	121	98	33·5	30·5	508	387	314	—	—	—	45·5	23	36·5	—	29	—	—	—	—
—	139·5	—	96·5	—	—	—	382	—	64·5	—	—	46	21	—	—	—	—	85·5	—	—
—	141·5	—	98	36·5	29·5	—	—	—	65·5	92	—	50	22·5	35·5	36·5	31	31	98	57	36
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	517	377	—	—	—	—	—	—	—	—	—	—	—	—	—
133	131·5	115	99	34	29·5	524	384	311	—	98	—	44	22	—	38	—	30	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	135·5	—	89·5	—	—	540	405	—	—	—	—	—	—	—	—	—	—	—	—	—
—	121	—	93	—	—	510	362	—	65·5	—	—	44·5	22·5	—	—	—	—	93·5	—	—
—	128·5	—	—	35	30·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	138	—	100	34	30·5	—	370	—	71·5	88·5	—	50	24	39	39·5	31·5	31·5	93·5	—	—

TABLE II.

G_1	G_2	W_1	W_2	h_1	f	B/L'	B/L	H'/L'	H/L	$G'H/GB$	NB/NH	O_2/O_1 L	O_2/O_1 R	G_2/G_1	P
—	—	—	—	33	43·5	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	69	—	—	—	—	—	—	—	—
—	—	—	—	33	45	—	—	—	—	—	—	—	—	—	—
—	—	—	115·5	35	46·5	—	—	—	—	—	—	—	—	—	—
55	—	—	—	41	46·5	—	69	—	—	—	44	—	—	—	—
—	—	—	—	—	—	71	71	72	72	—	43	88	89	—	—
53	—	—	—	—	—	72	72	71	71	65	45	82	83	—	86°
—	—	—	—	—	—	—	—	—	—	70	49	80	78	—	84°
—	—	—	—	—	—	73	72	71	70	—	45	89	87	—	—
—	—	123·5	—	38	42	—	—	—	—	—	45	—	—	—	—
56·5	41	—	—	—	—	—	—	—	—	—	48	—	93	73	—
—	—	111	105	37·5	45	—	75	—	—	78	45	84	—	—	—
—	—	—	99·5	31	46	—	66	—	—	—	—	—	—	—	—
55·5	—	—	—	33	45	73	73	69	69	76	41	85	87	—	81°
—	—	—	—	—	—	—	—	—	—	—	—	—	66	—	—
58	44	125·5	112	30·5	46·5	72	72	72	73	71	55	—	—	76	—
—	—	—	104	36·5	51·5	77	78	71	71	78	48	—	—	—	81°
—	—	—	97·5	32·5	43	—	80	—	77	—	—	—	—	—	—
51	43	126·5	105·5	31·5	45	—	77	—	72	—	60	77	78	84	—
57·5	47	121	99·5	35	46	80	81	78	79	67	61	—	76	82	83°
—	—	119·5	108	32	45·5	—	84	—	72	70	53	—	—	—	—
56	43·5	121·5	102·5	32·5	48	77	78	74	74	68	53	82	—	77	—
—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	81	—	—	—	—	—	—	—	—
—	—	—	—	—	—	74	73	75	74	—	50	85	83	—	—
—	—	—	—	—	—	75	75	78	77	—	51	79	—	—	—
—	—	—	—	—	—	—	75	—	79	—	46	—	—	—	—
57	36	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	75	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	70	—	—	—	—	—	—	—	—
—	—	117	86·5	30	47·5	74	74	71	71	—	50	—	79	—	—
—	—	104·5	85·5	28	43	—	74	—	—	—	—	—	—	—	—
—	—	117	97	—	39	—	—	—	—	—	—	—	—	—	—
—	—	—	96·5	37·5	47·5	—	76	—	72	—	—	—	—	—	—
—	—	—	89	30	37	—	76	—	68	—	50	—	—	—	—
—	—	—	—	—	—	—	86	—	76	—	—	—	—	—	—
—	—	—	103	33·5	42·5	—	81	—	75	81	48	81	80	—	—

Number of Skull	*L'*	*L*	*L''*	*F*	*B*	*B''*	*B'*	*H'*	*H*	*OH*	*LB*	*fml*	*fmb*	*U*	*S*	*Q*	*G'H*	*GB*	*J*
90	—	185	177	178·5	151·5	128	98	—	135·5	—	98·5	—	—	535	387	—	—	—	—
93	—	201	197	194	132	114·5	96·5	—	137	—	112	—	—	—	—	—	77	94	130
96	—	179·5	175·5	178·5	—	123	99·5	—	135·5	—	100·5	32·5	30·5	—	369	—	66	101	—
102	—	195·5	188	193	—	—	—	—	—	—	—	—	—	547	—	—	—	—	—
111	—	200	198·5	197·5	134	115·5	96·5	—	149	—	104·5	—	—	545	—	—	70	—	—
114	—	184	—	181·5	136	122·5	100·5	—	—	—	—	—	—	520	—	—	—	—	—
117	—	182	180·5	184·5	148	123·5	93	—	134·5	—	107·5	33	29	524	367	—	—	—	—
118	—	202·5	199	199	141	125	105·5	—	—	—	—	—	—	558	385	—	69	—	130
119	—	192·5	188·5	193·5	132·5	—	95	—	—	—	—	—	—	—	—	—	—	—	—
122	—	190	186	193	148·5	121·5	104·5	—	—	—	—	—	—	540	—	—	—	—	—
123	—	196·5	194·5	193·5	137	—	98·5	—	145·5	—	117·5	—	—	534	382	—	75·5	100·5	—
125	—	197·5	194·5	194·5	139	115·5	101	—	—	—	—	—	—	544	382	—	73	101·5	—
126	—	194	192	194	132·5	—	96·5	—	—	—	—	—	—	526	—	—	—	—	—
127	—	183	182	180	159	130	100	—	132	—	99	37	34	543	376·5	—	75	100	—
132	—	191	—	195·5	154	—	100·5	—	—	—	—	—	—	—	—	—	—	—	—
133	—	191·5	188	189	152·5	121·5	100·5	—	120	—	99	—	—	550	390	—	—	—	—
134	—	199	196·5	196	134·5	108·5	97	—	—	—	—	—	—	535	—	—	—	—	—
135	184	182·5	180·5	180	139	118·5	100·5	—	—	129	—	—	—	529	372	347	70	94·5	—
137	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
138	182	181	178	183	143·5	127	98	141	141	119	100·5	37·5	32·5	—	—	330	67	92	140
141	—	196	—	190·5	147·5	117·5	99·5	—	—	—	—	—	—	557	—	—	—	—	—
149	—	187·5	187	186·5	148	125·5	97·5	—	132·5	—	105	33·5	30	535	378	—	—	—	—
153	—	191·5	187·5	190	158·5	—	104	—	—	—	—	38	29	557	—	—	73	100	—
156	—	180	180	183·5	146	127	103	—	133·5	—	115·5	—	—	534	—	—	—	—	—
165	—	191	—	178	148	137	—	—	129	—	—	38·5	31·5	540	355	—	—	—	—
169	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
173	—	174	171	177	147	121·5	97	—	138	—	100	35	30·5	515	370	—	—	—	—
178	—	—	—	—	—	—	—	—	—	—	—	41	29·5	—	—	—	—	—	—
185	187	184	181	186	152	128	100·5	—	—	122	—	—	—	536	—	331	65·5	96	—
186	—	—	—	—	—	—	—	—	—	—	—	40·5	32	—	—	—	—	—	—
187	—	182	180	181·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
188	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
190	191	191·5	186·5	190	147	128·5	99·5	141	141	120	106·5	—	—	545	382	328	71·5	98·5	—
191	—	—	179·5	182	—	—	—	—	—	—	—	38	33	—	—	—	—	—	—
193	189	189	184·5	185·5	137	—	100·5	—	—	128	—	39	32·5	532	—	335	—	—	—
195	189	187	185	187	138·5	116	91·5	132	132	120	95·5	37·5	31·5	527	380·5	321	70·5	90·5	—
196	—	—	—	—	—	—	—	—	—	—	—	39	33	—	—	—	—	—	—
197	—	183	177·5	183	—	—	—	—	—	—	100·5	—	—	—	—	—	69	—	—
207	183	185·5	180	184	151·5	126	99	—	—	126	—	34·5	26·5	—	—	335	—	—	—
214	—	185·5	180·5	186·5	153·5	123·5	101·5	—	—	—	—	—	—	542	—	—	—	—	—
215	—	182	—	185	—	—	92	—	—	—	—	—	—	—	—	—	—	—	—
217	—	191·5	189·5	185	150	—	97	—	—	—	—	—	—	550	—	—	66	100·5	—
218	181	181	177	180	149	117	100·5	—	139	125	101	—	—	522	372	340	69	—	—
222	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

ROUND BARROW. ♂.

MEASUREMENTS

G'H	*GB*	*J*	*NH*	*NB*	O_1 *L*	O_1 *R*	O_2 *L*	O_2 *R*	*GL*	G_1	G_2	W_1	W_2	h_1	*f*	*B/L'*	*B/L*
—	—	—	—	—	—	—	—	—	—	—	—	—	106·5	—	42	—	82
77	94	130	56	25·5	—	—	—	—	102·5	—	38	—	—	—	—	—	66
66	101	—	50	25·5	37·5	38	27	26·5	96	52	37·5	123	93	37	42·5	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
70	—	—	—	—	—	—	—	—	—	—	39·5	—	—	29·5	44	—	67
—	—	—	—	—	—	—	—	—	—	—	—	122	107	36·5	49·5	—	74
—	—	—	—	—	35·5	36·5	34	34·5	—	—	—	105·5	93·5	—	—	—	81
69	—	130	48	26·5	—	39·5	—	29	—	62·5	38·5	—	109	—	44	—	70
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	69
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75·5	100·5	—	51·5	25	38·5	39·5	33	32·5	105	54	42·5	119	101·5	35·5	43	—	71
73	101·5	—	48	24	—	—	—	—	—	58·5	40·5	—	—	37	47·5	—	72
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	68
75	100	—	57·5	26	38	38·5	31·5	30	—	57	46·5	129	107	34	46·5	—	87
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	81
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	80
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	67
70	94·5	—	46	23·5	39	38·5	—	30·5	—	—	—	—	93	36	—	76	76
—	—	—	—	—	—	—	—	—	—	—	—	135	101·5	32	46·5	—	—
67	92	140·5	50	22·5	—	40·5	—	32·5	91·5	55·5	44	—	107	32·5	—	79	79
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	75
—	—	—	—	—	40	40	32·5	32	—	—	—	—	—	—	—	—	79
73	100	—	50	23·5	39	41·5	34·5	33·5	—	57·5	42	130	119	36·5	49	—	83
—	—	—	—	—	—	—	—	—	—	—	—	—	107·5	32·5	—	—	81
—	—	—	—	—	—	46·5	—	32·5	—	—	—	—	—	—	—	—	77
—	—	—	—	—	—	—	—	—	—	—	—	—	115·5	35	51·5	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	84
—	—	—	—	—	—	40·5	—	33·5	—	—	—	129·5	110·5	34	44	—	—
65·5	96	—	48	25	39	38·5	31	31·5	—	—	46	—	103·5	32·5	45·5	81	83
—	—	—	—	—	40	40	31	30·5	—	—	—	126·5	105·5	35·5	42	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	117·5	34	45	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	30	43·5	—	—
71·5	98·5	—	—	—	—	42·5	—	32	94	55·5	41	117	102·5	31·5	—	77	77
—	—	—	—	—	—	40·5	—	32	—	—	—	—	—	—	—	—	—
—	—	—	—	—	40·5	40·5	32·5	32	—	—	—	129	98·5	32·5	43·5	72	72
70·5	90·5	—	51	21·5	—	—	—	—	92·5	54·5	37	120·5	89	—	41	73	74
—	—	—	—	—	41	—	35·5	—	—	—	—	117·5	98·5	—	49·5	—	—
69	—	—	51	27	39	39	33	32	94·5	51·5	44·5	—	—	31	—	—	—
—	—	—	—	—	43	43	33·5	34	—	—	—	—	122	35·5	45	83	82
—	—	—	—	—	—	40	—	32·5	—	—	—	—	110	—	41·5	—	83
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
66	100·5	—	46	25	39	39·5	32·5	34	—	—	37·5	—	—	—	—	—	79
69	—	—	49·5	24	36·5	—	33	—	92	53	36	—	—	—	—	82	82
—	—	—	—	—	—	37	—	30	—	—	—	—	102·5	—	48·5	—	—

ROUND BARROW. ♂.

MEASUREMENTS

LB	*fml*	*fmb*	*U*	*S*	*Q*	*G'H*	*GB*	*J*	*NH*	*NB*	O_1 *L*	O_1 *R*	O_2 *L*	O_2 *R*	*GL*	G_1	G_2	W_1	W_2	
98·5	—	—	535	387	—	—	—	—	—	—	—	—	—	—	—	—	—	—	106·5	
112	—	—	—	—	—	77	94	130	56	25·5	—	—	—	—	102·5	—	38	—	—	
100·5	32·5	30·5	—	369	—	66	101	—	50	25·5	37·5	38	27	26·5	96	52	37·5	123	93	3
—	—	—	547	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
104·5	—	—	545	—	—	70	—	—	—	—	—	—	—	—	—	—	39·5	—	—	2
—	—	—	520	—	—	—	—	—	—	—	—	—	—	—	—	—	—	122	107	3
107·5	33	29	524	367	—	—	—	—	—	—	35·5	36·5	34	34·5	—	—	—	105·5	93·5	
—	—	—	558	385	—	69	—	130	48	26·5	—	39·5	—	29	—	62·5	38·5	—	109	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	—	—	540	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
117·5	—	—	534	382	—	75·5	100·5	—	51·5	25	38·5	39·5	33	32·5	105	54	42·5	119	101·5	3
—	—	—	544	382	—	73	101·5	—	48	24	—	—	—	—	—	58·5	40·5	—	—	3
—	—	—	526	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
99	37	34	543	376·5	—	75	100	—	57·5	26	38	38·5	31·5	30	—	57	46·5	129	107	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
99	—	—	550	390	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	—	—	535	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	—	—	529	372	347	70	94·5	—	46	23·5	39	38·5	—	30·5	—	—	—	—	93	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	135	101·5	3
100·5	37·5	32·5	—	—	330	67	92	140·5	50	22·5	—	40·5	—	32·5	91·5	55·5	44	—	107	3
—	—	—	557	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
105	33·5	30	535	378	—	—	—	—	—	—	40	40	32·5	32	—	—	—	—	—	
—	38	29	557	—	—	73	100	—	50	23·5	39	41·5	34·5	33·5	—	57·5	42	130	119	3
115·5	—	—	534	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	107·5	3
—	38·5	31·5	540	355	—	—	—	—	—	—	—	46·5	—	32·5	—	—	—	—	—	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	115·5	3
100	35	30·5	515	370	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	41	29·5	—	—	—	—	—	—	—	—	—	40·5	—	33·5	—	—	—	129·5	110·5	3
—	—	—	536	—	331	65·5	96	—	48	25	39	38·5	31	31·5	—	—	46	—	103·5	3
—	40·5	32	—	—	—	—	—	—	—	—	40	40	31	30·5	—	—	—	126·5	105·5	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	117·5	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
106·5	—	—	545	382	328	71·5	98·5	—	—	—	—	42·5	—	32	94	55·5	41	117	102·5	3
—	38	33	—	—	—	—	—	—	—	—	—	40·5	—	32	—	—	—	—	—	
—	39	32·5	532	—	335	—	—	—	—	—	40·5	40·5	32·5	32	—	—	—	129	98·5	3
95·5	37·5	31·5	527	380·5	321	70·5	90·5	—	51	21·5	—	—	—	—	92·5	54·5	37	120·5	89	
—	39	33	—	—	—	—	—	—	—	—	41	—	35·5	—	—	—	—	117·5	98·5	
100·5	—	—	—	—	—	69	—	—	51	27	39	39	33	32	94·5	51·5	44·5	—	—	3
—	34·5	26·5	—	—	335	—	—	—	—	—	43	43	33·5	34	—	—	—	—	122	3
—	—	—	542	—	—	—	—	—	—	—	—	40	—	32·5	—	—	—	—	110	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	—	—	550	—	—	66	100·5	—	46	25	39	39·5	32·5	34	—	—	37·5	—	—	
101	—	—	522	372	340	69	—	—	49·5	24	36·5	—	33	—	92	53	36	—	—	
—	—	—	—	—	—	—	—	—	—	—	—	37	—	30	—	—	—	—	102·5	

TABLE III.

W_1	W_2	h_1	f	B/L'	B/L	H'/L'	H/L	$G'H/GB$	NB/NH	O_2/O_1 L	O_2/O_1 R	G_2/G_1	P
—	106·5	—	42	—	82	—	73	—	—	—	—	—	—
—	—	—	—	—	66	—	68	82	46	—	—	—	—
123	93	37	42·5	—	—	—	75	65	51	72	70	72	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	29·5	44	—	67	—	74	—	—	—	—	—	—
122	107	36·5	49·5	—	74	—	—	—	—	—	—	—	—
105·5	93·5	—	—	—	81	—	74	—	—	96	95	—	—
—	109	—	44	—	70	—	—	—	55	—	73	62	—
—	—	—	—	—	69	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
119	101·5	35·5	43	—	71	—	74	75	49	86	82	79	—
—	—	37	47·5	—	72	—	—	72	50	—	—	69	—
—	—	—	—	—	68	—	—	—	—	—	—	—	—
129	107	34	46·5	—	87	—	72	75	45	83	78	82	—
—	—	—	—	—	81	—	—	—	—	—	—	—	—
—	—	—	—	—	80	—	63	—	—	—	—	—	—
—	—	—	—	—	67	—	—	—	—	—	—	—	—
—	93	36	—	76	76	—	—	74	51	79	—	—	—
135	101·5	32	46·5	—	—	—	—	—	—	—	—	—	—
—	107	32·5	—	79	79	77	78	73	45	—	80	79	82
—	—	—	—	—	75	—	—	—	—	—	—	—	—
—	—	—	—	—	79	—	71	—	—	81	80	—	—
130	119	36·5	49	—	83	—	—	73	47	88	81	73	—
—	107·5	32·5	—	—	81	—	74	—	—	—	—	—	—
—	—	—	—	—	77	—	68	—	—	—	70	—	—
—	115·5	35	51·5	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	84	—	79	—	—	—	—	—	—
129·5	110·5	34	44	—	—	—	—	—	—	—	83	—	—
—	103·5	32·5	45·5	81	83	—	—	68	52	78	82	—	85
126·5	105·5	35·5	42	—	—	—	—	—	—	77	76	—	84
—	117·5	34	45	—	—	—	—	—	—	—	—	—	—
—	—	30	43·5	—	—	—	—	—	—	—	—	—	—
117	102·5	31·5	—	77	77	74	74	73	—	—	75	74	85
—	—	—	—	—	—	—	—	—	—	—	79	—	—
129	98·5	32·5	43·5	72	72	—	—	—	—	80	79	—	—
120·5	89	—	41	73	74	70	71	78	42	—	—	68	82
117·5	98·5	—	49·5	—	—	—	—	—	—	87	—	—	—
—	—	31	—	—	—	—	—	—	53	85	82	86	—
—	122	35·5	45	83	82	—	—	—	—	78	79	—	82
—	110	—	41·5	—	83	—	—	—	—	—	81	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	79	—	—	66	54	83	86	—	—
—	—	—	—	82	82	—	77	—	48	90	—	68	89
—	102·5	—	48·5	—	—	—	—	—	—	—	81	—	—

ROUND BARROW. ♂.

MEASUREMENTS

LB	*fml*	*fmb*	*U*	*S*	*Q*	*G'H*	*GB*	*J*	*NH*	*NB*	O_1 *L*	O_1 *R*	O_2 *L*	O_2 *R*	*GL*	G_1	G_2	W_1	W_2	
98·5	—	—	535	387	—	—	—	—	—	—	—	—	—	—	—	—	—	—	106·5	
112	—	—	—	—	—	77	94	130	56	25·5	—	—	—	—	102·5	—	38	—	—	
100·5	32·5	30·5	—	369	—	66	101	—	50	25·5	37·5	38	27	26·5	96	52	37·5	123	93	3
—	—	—	547	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
104·5	—	—	545	—	—	70	—	—	—	—	—	—	—	—	—	—	39·5	—	—	2
—	—	—	520	—	—	—	—	—	—	—	—	—	—	—	—	—	—	122	107	3
107·5	33	29	524	367	—	—	—	—	—	—	35·5	36·5	34	34·5	—	—	—	105·5	93·5	
—	—	—	558	385	—	69	—	130	48	26·5	—	39·5	—	29	—	62·5	38·5	—	109	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	—	—	540	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
117·5	—	—	534	382	—	75·5	100·5	—	51·5	25	38·5	39·5	33	32·5	105	54	42·5	119	101·5	3
—	—	—	544	382	—	73	101·5	—	48	24	—	—	—	—	—	58·5	40·5	—	—	3
—	—	—	526	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
99	37	34	543	376·5	—	75	100	—	57·5	26	38	38·5	31·5	30	—	57	46·5	129	107	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
99	—	—	550	390	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	—	—	535	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	—	—	529	372	347	70	94·5	—	46	23·5	39	38·5	—	30·5	—	—	—	—	93	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	135	101·5	3
100·5	37·5	32·5	—	—	330	67	92	140·5	50	22·5	—	40·5	—	32·5	91·5	55·5	44	—	107	3
—	—	—	557	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
105	33·5	30	535	378	—	—	—	—	—	—	40	40	32·5	32	—	—	—	—	—	
—	38	29	557	—	—	73	100	—	50	23·5	39	41·5	34·5	33·5	—	57·5	42	130	119	3
115·5	—	—	534	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	107·5	3
—	38·5	31·5	540	355	—	—	—	—	—	—	—	46·5	—	32·5	—	—	—	—	—	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	115·5	3
100	35	30·5	515	370	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	41	29·5	—	—	—	—	—	—	—	—	—	40·5	—	33·5	—	—	—	129·5	110·5	3
—	—	—	536	—	331	65·5	96	—	48	25	39	38·5	31	31·5	—	—	46	—	103·5	3
—	40·5	32	—	—	—	—	—	—	—	—	40	40	31	30·5	—	—	—	126·5	105·5	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	117·5	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
106·5	—	—	545	382	328	71·5	98·5	—	—	—	—	42·5	—	32	94	55·5	41	117	102·5	3
—	38	33	—	—	—	—	—	—	—	—	—	40·5	—	32	—	—	—	—	—	
—	39	32·5	532	—	335	—	—	—	—	—	40·5	40·5	32·5	32	—	—	—	129	98·5	3
95·5	37·5	31·5	527	380·5	321	70·5	90·5	—	51	21·5	—	—	—	—	92·5	54·5	37	120·5	89	
—	39	33	—	—	—	—	—	—	—	—	41	—	35·5	—	—	—	—	117·5	98·5	
100·5	—	—	—	—	—	69	—	—	51	27	39	39	33	32	94·5	51·5	44·5	—	—	3
—	34·5	26·5	—	—	335	—	—	—	—	—	43	43	33·5	34	—	—	—	—	122	3
—	—	—	542	—	—	—	—	—	—	—	—	40	—	32·5	—	—	—	—	110	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
—	—	—	550	—	—	66	100·5	—	46	25	39	39·5	32·5	34	—	—	37·5	—	—	
101	—	—	522	372	340	69	—	—	49·5	24	36·5	—	33	—	92	53	36	—	—	
—	—	—	—	—	—	—	—	—	—	—	—	37	—	30	—	—	—	—	102·5	

TABLE III.

W_1	W_2	h_1	f	B/L'	B/L	H'/L'	H/L	$G'H/GB$	NB/NH	O_2/O_1 L	O_2/O_1 R	G_2/G_1	P
—	106·5	—	42	—	82	—	73	—	—	—	—	—	—
—	—	—	—	—	66	—	68	82	46	—	—	—	—
123	93	37	42·5	—	—	—	75	65	51	72	70	72	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	29·5	44	—	67	—	74	—	—	—	—	—	—
122	107	36·5	49·5	—	74	—	—	—	—	—	—	—	—
105·5	93·5	—	—	—	81	—	74	—	—	96	95	—	—
—	109	—	44	—	70	—	—	—	55	—	73	62	—
—	—	—	—	—	69	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
119	101·5	35·5	43	—	71	—	74	75	49	86	82	79	—
—	—	37	47·5	—	72	—	—	72	50	—	—	69	—
—	—	—	—	—	68	—	—	—	—	—	—	—	—
129	107	34	46·5	—	87	—	72	75	45	83	78	82	—
—	—	—	—	—	81	—	—	—	—	—	—	—	—
—	—	—	—	—	80	—	63	—	—	—	—	—	—
—	—	—	—	—	67	—	—	—	—	—	—	—	—
—	93	36	—	76	76	—	—	74	51	79	—	—	—
135	101·5	32	46·5	—	—	—	—	—	—	—	—	—	—
—	107	32·5	—	79	79	77	78	73	45	—	80	79	82°
—	—	—	—	—	75	—	—	—	—	—	—	—	—
—	—	—	—	—	79	—	71	—	—	81	80	—	—
130	119	36·5	49	—	83	—	—	73	47	88	81	73	—
—	107·5	32·5	—	—	81	—	74	—	—	—	—	—	—
—	—	—	—	—	77	—	68	—	—	—	70	—	—
—	115·5	35	51·5	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	84	—	79	—	—	—	—	—	—
129·5	110·5	34	44	—	—	—	—	—	—	—	83	—	—
—	103·5	32·5	45·5	81	83	—	—	68	52	78	82	—	85°
126·5	105·5	35·5	42	—	—	—	—	—	—	77	76	—	84°
—	117·5	34	45	—	—	—	—	—	—	—	—	—	—
—	—	30	43·5	—	—	—	—	—	—	—	—	—	—
117	102·5	31·5	—	77	77	74	74	73	—	—	75	74	85°
—	—	—	—	—	—	—	—	—	—	—	79	—	—
129	98·5	32·5	43·5	72	72	—	—	—	—	80	79	—	—
120·5	89	—	41	73	74	70	71	78	42	—	—	68	82°
117·5	98·5	—	49·5	—	—	—	—	—	—	87	—	—	—
—	—	31	—	—	—	—	—	—	53	85	82	86	—
—	122	35·5	45	83	82	—	—	—	—	78	79	—	82°
—	110	—	41·5	—	83	—	—	—	—	—	81	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	79	—	—	66	54	83	86	—	—
—	—	—	—	82	82	—	77	—	48	90	—	68	89°
—	102·5	—	48·5	—	—	—	—	—	—	—	81	—	—

Number of Skull	*L'*	*L*	*L''*	*F*	*B*	*B''*	*B'*	*H'*	*H*	*OH*	*LB*	*fml*	*fmb*	*U*	*S*	*Q*	*G'H*	*GB*	*J*
91	—	185	183·5	180	143	—	100	—	—	—	—	—	—	528	—	—	—	—	—
94	—	184	182	181·5	132·5	110	97	—	137	—	102·5	33	29	513	372	—	68·5	—	—
98	—	171	167	173·5	138	111	96	—	127	—	95	36·5	30	464	345	—	—	—	—
99	180	181·5	180	184·5	124	106·5	86·5	134	133	112	96·5	—	—	465	368	288	70	89	—
100	—	185	—	192·5	136	—	—	—	—	—	—	—	—	525	—	—	—	—	—
103	—	169	167	170·5	138	—	98	—	—	—	—	—	—	486	—	—	—	—	—
104	169	163	162·5	170	143·5	116·5	93	—	—	116	—	—	—	498	350	316	68	86·5	—
105	—	187	184	184	136·5	113·5	97	—	—	—	—	—	—	529	—	—	65·5	—	—
110	166	166·5	166	168	140	—	93·5	—	—	117	—	—	—	—	—	317	—	—	—
112	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
113	—	170	171	174·5	136·5	—	97	—	131·5	91	—	—	—	498	370	—	—	—	—
115	—	187·5	184	186·5	138·5	109	—	—	—	—	—	—	—	—	365	—	—	—	—
121	—	175·5	175	180	141	118	100	—	126·5	—	92	40	29·5	517	369	—	—	—	—
130	—	174·5	172·5	173	140	116·5	97·5	—	—	—	—	—	—	505	—	—	—	—	—
136	—	169	165·5	168·5	139·5	112	101	—	128	—	93	—	—	498	348	—	—	—	—
142	187	186	185	185·5	134·5	109·5	99	—	142	117	108·5	34·5	—	521	375	314	74	—	—
143	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
144	183	183	181·5	182	138·5	112	94	138	136·5	121	114·5	30	28	508	375	314	62	89	—
148	—	—	—	174·5	133·5	124	87	—	124	—	—	—	—	498	—	—	—	—	—
150	—	178·5	176	181·5	136	109·5	96	—	126·5	—	95·5	—	—	505	373	—	—	—	—
151	—	190·5	189	189·5	132	101·5	95	—	—	—	—	—	—	524	380	—	—	—	—
159	—	175·5	175	173·5	137	—	—	—	—	—	—	—	—	502	—	—	—	—	—
160	—	175	171	175·5	137	113·5	92·5	—	139	—	97	34	26·5	499	364	—	63	88·5	—
162	192	188·5	185	189	143	—	97·5	—	—	119	—	—	—	542	—	332	—	—	—
166	—	179	175·5	181	148	—	95·5	—	—	—	—	—	—	523	—	—	—	—	—
171	178	178·5	176	177·5	132	119	99	—	111	—	—	—	—	508	—	294	62	90	—
177	184	184	180	183	138·5	116·5	95	—	—	118	—	—	—	518	—	316	—	—	—
180	—	175	174·5	172·5	142·5	114	100	—	140·5	—	100	37·5	28	504	367	—	65·5	90	—
192	178	178	176·5	178·5	138·5	116	99	—	—	123	—	—	—	512	—	324	—	—	—
199	—	167	166	169	141·5	—	92	—	—	—	—	—	—	490	—	—	—	—	—
200	176	175	174·5	176·5	136	120	—	—	—	121	—	—	—	500	—	312	—	—	—
201	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
202	—	173·5	172	175	—	—	—	—	136·5	—	102·5	39·5	30·5	—	—	—	63·5	—	—
204	—	183·5	—	186	128	—	99·5	—	—	—	—	—	—	513	—	—	—	—	—
206	—	182	—	184·5	149	—	93	—	—	—	—	—	—	—	—	—	—	—	—
211	—	168	—	173	137·5	—	—	—	—	—	—	—	—	483	—	—	—	—	—
212	181	179·5	177·5	177·5	139·5	113	100	—	—	120	—	—	—	517	356	322	—	99	—
221	—	182	181·5	180	143·5	118·5	93	—	123	—	95	34	—	—	—	—	—	—	—

ROUND BARROW. ♀.

MEASUREMENTS

'H	GB	J	NH	NB	O_1 L	O_1 R	O_2 L	O_2 R	GL	G_1	G_3	W_1	W_2	h_1	f	B|L'	B|L	H'|L'	H|L
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	77	—	—
8·5	—	—	44·5	24	—	39	—	33·5	98·5	53·5	37·5	107·5	92·5	34·5	45	—	72	—	74
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	81	—	74
0	89	—	52·5	25	37·5	38·5	33	33·5	83	48	37	—	86	33·5	39	69	69	74	73
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	74	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	82	—	—
8	86·5	—	49·5	22	—	—	—	—	—	51·5	39	—	—	—	—	85	88	—	—
5·5	—	—	48	23·5	—	40·5	—	31	—	—	40·5	—	—	31·5	43	—	73	—	—
—	—	—	—	—	—	40·5	—	33	—	—	—	117·5	98	26	44	84	84	—	—
—	—	—	—	—	—	—	—	—	—	—	—	123·5	—	27·5	41·5	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	80	—	77
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	74	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	97·5	26·5	43·5	—	81	—	72
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	80	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	82	—	76
4	—	—	52	22	36·5	—	31·5	—	—	54	40·5	—	—	—	—	72	72	—	76
—	—	—	—	—	—	—	—	—	—	—	—	—	103·5	33	45	—	—	—	—
2	89	—	—	—	38·5	39	33·5	—	104	—	—	112	93·5	32	45·5	75	75	75	74
—	—	—	—	—	—	—	—	—	—	—	—	—	100·5	34·5	46	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	76	—	71
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	69	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	78	—	—
3	88·5	—	46·5	23·5	36·5	36·5	32	31	86	49	36	—	92	30	45	—	78	—	79
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	74	76	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	83	—	—
2	90	—	48	24·5	—	41	—	31·5	92	—	—	123	95	27	—	74	74	—	—
—	—	—	—	—	40	—	33	—	—	—	—	123	99·5	32	43	75	75	—	—
5·5	90	—	43	22·5	39·5	40·5	32·5	—	98·5	49	41·5	115	86	36·5	43·5	—	81	—	80
—	—	—	—	—	41	42	32	33	—	—	—	123·5	—	29·5	41·5	78	78	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	43	—	84	—	—
—	—	—	—	—	—	36	—	31	—	—	—	—	—	31·5	—	77	78	—	—
—	—	—	—	—	41·5	—	33	—	—	—	—	113	89	29·5	46	—	—	—	—
3·5	—	—	50	24·5	—	—	—	—	91	49·5	41·5	—	—	—	—	—	—	—	79
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	70	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	82	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	82	—	—
—	99	—	—	—	38	—	33·5	—	—	51	38·5	115·5	100	30	44·5	77	78	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	79	—	68

ROUND BARROW. ♀.

MEASUREMENTS

OH	*LB*	*fml*	*fmb*	*U*	*S*	*Q*	*G'H*	*GB*	*J*	*NH*	*NB*	O_1 *L*	O_1 *R*	O_2 *L*	O_2 *R*	*GL*	G_1	G_2	W_1
—	—	—	—	528	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	102·5	33	29	513	372	—	68·5	—	—	44·5	24	—	39	—	33·5	98·5	53·5	37·5	107
—	95	36·5	30	464	345	—	—	—	—	—	—	—	—	—	—	—	—	—	—
112	96·5	—	—	465	368	288	70	89	—	52·5	25	37·5	38·5	33	33·5	83	48	37	—
—	—	—	—	525	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	486	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
116	—	—	—	498	350	316	68	86·5	—	49·5	22	—	—	—	—	—	51·5	39	—
—	—	—	—	529	—	—	65·5	—	—	48	23·5	—	40·5	—	31	—	—	40·5	—
117	—	—	—	—	—	317	—	—	—	—	—	—	40·5	—	33	—	—	—	117
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	123
91	—	—	—	498	370	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	365	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	92	40	29·5	517	369	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	505	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	93	—	—	498	348	—	—	—	—	—	—	—	—	—	—	—	—	—	—
117	108·5	34·5	—	521	375	314	74	—	—	52	22	36·5	—	31·5	—	—	54	40·5	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
121	114·5	30	28	508	375	314	62	89	—	—	—	38·5	39	33·5	—	104	—	—	112
—	—	—	—	498	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	95·5	—	—	505	373	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	524	380	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	502	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	97	34	26·5	499	364	—	63	88·5	—	46·5	23·5	36·5	36·5	32	31	86	49	36	—
119	—	—	—	542	—	332	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	523	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	508	—	294	62	90	—	48	24·5	—	41	—	31·5	92	—	—	123
118	—	—	—	518	—	316	—	—	—	—	—	40	—	33	—	—	—	—	123
—	100	37·5	28	504	367	—	65·5	90	—	43	22·5	39·5	40·5	32·5	—	98·5	49	41·5	115
123	—	—	—	512	—	324	—	—	—	—	—	41	42	32	33	—	—	—	123
—	—	—	—	490	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
121	—	—	—	500	—	312	—	—	—	—	—	—	36	—	31	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	41·5	—	33	—	—	—	—	113
—	102·5	39·5	30·5	—	—	—	63·5	—	—	50	24·5	—	—	—	—	91	49·5	41·5	—
—	—	—	—	513	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	483	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
120	—	—	—	517	356	322	—	99	—	—	—	38	—	33·5	—	—	51	38·5	115
—	95	34	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE IV.

G_1	G_2	W_1	W_2	h_1	f	B/L'	B/L	H'/L'	H/L	$G'H/GB$	NB/NH	$\frac{O_2}{L}/O_1$	$\frac{O_2}{R}/O_1$	G_2/G_1	P
—	—	—	—	—	—	—	77	—	—	—	—	—	—	—	—
3·5	37·5	107·5	92·5	34·5	45	—	72	—	74	—	54	—	86	70	—
—	—	—	—	—	—	—	81	—	74	—	—	—	—	—	—
8	37	—	86	33·5	39	69	69	74	73	79	48	88	87	77	84°
—	—	—	—	—	—	—	74	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	—
1·5	39	—	—	—	—	85	88	—	—	79	44	—	—	75	85°
—	40·5	—	—	31·5	43	—	73	—	—	—	49	—	76	—	—
—	—	117·5	98	26	44	84	84	—	—	—	—	—	81	—	86°
—	—	123·5	—	27·5	41·5	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	80	—	77	—	—	—	—	—	—
—	—	—	—	—	—	—	74	—	—	—	—	—	—	—	—
—	—	—	97·5	26·5	43·5	—	81	—	72	—	—	—	—	—	—
—	—	—	—	—	—	—	80	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	82	—	76	—	—	—	—	—	—
4	40·5	—	—	—	—	72	72	—	76	—	42	86	—	75	84°
—	—	—	103·5	33	45	—	—	—	—	—	—	—	—	—	—
—	—	112	93·5	32	45·5	75	75	75	74	70	—	87	—	—	75°
—	—	—	100·5	34·5	46	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	76	—	71	—	—	—	—	—	—
—	—	—	—	—	—	—	69	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	78	—	—	—	—	—	—	—	—
9	36	—	92	30	45	—	78	—	79	71	51	88	85	73	—
—	—	—	—	—	—	74	76	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	83	—	—	—	—	—	—	—	—
—	—	123	95	27	—	74	74	—	—	69	51	—	77	—	—
—	—	123	99·5	32	43	75	75	—	—	—	—	82	—	—	—
9	41·5	115	86	36·5	43·5	—	81	—	80	73	52	82	—	85	90°
—	—	123·5	—	29·5	41·5	78	78	—	—	—	—	78	79	—	—
—	—	—	—	—	43	—	84	—	—	—	—	—	—	—	—
—	—	—	—	31·5	—	77	78	—	—	—	—	—	86	—	—
—	—	113	89	29·5	46	—	—	—	—	—	—	79	—	—	—
9·5	41·5	—	—	—	—	—	—	—	79	—	49	—	—	84	—
—	—	—	—	—	—	—	70	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	82	—	—	—	—	—	—	—	—
1	38·5	115·5	100	30	44·5	77	78	—	—	—	—	88	—	75	—
—	—	—	—	—	—	—	79	—	68	—	—	—	—	—	—

DESCRIPTIVE CATALOGUE.

In the accompanying remarks:

Calvaria = Dome = Schädeldach.
Cranium = Braincase.
Skull = Braincase and face bones.
ad. = Adult.
B. B. = Greenwell's *British Barrows*.
G. C. = Greenwell Collection in Oxford Museum.

The page of the *British Barrows*, together with the line on the page in which the particular skull is referred to are, when possible, cited. The Roman numerals give the number of the special excavation in the same work.

LONG BARROWS (MALE).

No. 3.—Calvaria, *ad.* Mandible without ascending portion of left ramus. Alveolar and palatal portions of maxillae, also left occipital condyle in box. Posterior sagittal depression. B. B. p. 484, ccxxi. Ebberston, N. R. Yorkshire. G. C.

No. 4.—Calvaria, old *ad.* Torus occipitalis. B. B. p. 484, ccxxi. Ebberston, N. R. Yorkshire. G. C. Cf. also MS. Catalogue, Oxford.

No. 9.—Imperfect Calvaria, *ad.* Mandible without ascending portion of right ramus; the left coronary process broken. Maxillae detached (*query* from same individual?). Supra-inial depression. B. B. p. 509, ccxxvii. Over Silton, N. R. Yorkshire. G. C. Cf. MS. Catalogue, Oxford.

No. 18.—Skull, *ad.* The right face-bones and the basi-cranial axis wanting, the mandible imperfect. Distorted by pressure. B. B. pp. 497—501, p. 501, l. 1, ccxxiv. Rudstone, E. R. Yorkshire. G. C.

No. 19.—Skull, old *ad.* Imperfect: basis cranii missing: mandible with most of the right ramus missing. B. B. p. 501, l. 6, and pp. 612 *et seq.* (figured and measured). Rudstone, E. R. Yorkshire. G. C.

No. 21.—Skull, old *ad.* Right parietal and squamosal imperfect. Thurnam, *Mem. Anthrop. Soc.* I. p. 132, p. 478. Dinnington, near Rotherham, W. R. Yorkshire.

No. 22.—Skull, old *ad.* Occipital imperfect. Metopic suture. Thurnam, *loc. cit.* Dinnington, near Rotherham, W. R. Yorkshire.

No. 25.—Skull, *adolescent.* Frontal defective (from abscess?); left palate broken. Paired precondylar eminences, short torus palatinus. Thurnam, *loc. cit.* Dinnington, near Rotherham, W. R. Yorkshire.

No. 38.—Skull, *ad.* Ossicles in lambdoid; vestiges of transverse occipital suture; left infra-orbital suture in face. *Crania britannica*, No. lxviii. Vol. I. pp. 244, 245, Vol. II. pp. 59 (3). Rodmarton, Gloucestershire. Thurnam Collection, No. 163.

No. 40.—Skull, young *ad.* Very defective; left temporal and basis cranii absent. Metopic suture. *Crania britannica*, No. lxx. Vol. I. pp. 244, 245, figured, Vol. II. 59 (4). Rodmarton, Gloucestershire. Thurnam Collection, No. 166.

No. 48.—Skull with mandible, *ad.* Sphenoid, palatine region, much of right frontal and parietal absent. Mandible without angle of right ramus. Rolleston, *Journ. Anthrop. Inst.* v. p. 151 (1875). Nether Swell, Gloucestershire.

No. 59.—Skull with mandible, *ad.* Basis cranii wanting: other extensive defects. Slight torus occipitalis; minute ossicles in lambdoid. B. B. p. 521, ccxxx. Rolleston, *loc. cit.* p. 169. Swell, Gloucestershire.

No. 62.—Skull with mandible, young *ad.* Very fragmentary. Mandible wanting both condyles, and not belonging to skull. B. B. pp. 524—527, l. 37, ccxxxii. Upper Swell, Gloucestershire.

No. 65.—Skull with mandible, *adolescent.* Mandible with greater part of right ramus wanting. Metopic suture; right infra-orbital suture on face, left *m* 3 just appearing. B. B. p. 529, l. 5. Upper Swell, Gloucestershire.

No. 72.—Imperfect calvaria, *ad.* Part of right side of face and temporal bone attached. B. B. p. 531, l. 11. Upper Swell, Gloucestershire.

No. 77.—Skull with mandible, *ad.* Much broken. G. Y. Ackerman, *Archaeologia*, xxxvii. p. 432. Thurnam, *Archaeologia*, xlii. Crawley, near Witney, Oxfordshire.

No. 80.—Skull with mandible, young *ad.* Mandible broken at symphysis. Ossicles in lambdoid: slight bathrocephaly. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 81.—Calvaria, with separate maxillae and mandible, *ad.* Mandible with right condyle broken. Metopic suture. Trace of torus occipitalis. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 82.—Skull with mandible, *ad.* Defects. Right canine displaced in front of lateral incisor; mastoids very small. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 83.—Skull with mandible, *ad.* (young ?). Right zygoma broken. Ossicles of pterion; the right 30 × 10 mm., the left 20 × 7 mm. Two ossicles in left coronal suture, the larger 20 × 20 mm. Ossicles of lambdoid, two on right, three on left side, the largest 20 × 15 mm. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 86.—Skull with mandible, *ad.* Skull much broken. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 87.—Skull, *ad.* Basis cranii and most of face wanting. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

LONG BARROW (IMMATURE, OR SEX DOUBTFUL).

No. 7.—Calvaria (♀ ?). Very imperfect. B. B. p. 484, ccxxi. Ebberston, N. R. Yorkshire. G. C.

No. 23.—Skull (sex ?). Thurnam, *loc. cit.* No. 21. Dinnington near Rotherham, W. R. Yorkshire.

No. 29.—Skull (♀ ?), *ad.* Left temporal and left side of face wanting. Very long mastoid. Thurnam, *loc. cit.* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 51.—Calvaria with mandible. Child 6 years. Metopic suture. Rolleston, *loc. cit. sub* No. 48, p. 156, l. 7, and p. 158, l. 21. Swell, Gloucestershire.

No. 54.—Skull with mandible. Face and basis cranii very imperfect. Rolleston, *loc. cit. sub* No. 48, p. 161. Swell, Gloucestershire.

No. 66.—Calvaria with left temporal (♂ ?), *ad.* Slight sagittal ridge; torus occipitalis; ossicles of lambdoid. B. B. p. 530. Upper Swell, Gloucestershire.

LONG BARROW (FEMALE).

No. 13.—Calvaria, *ad.* Imperfect: additional fragments. B. B. pp. 491, 492, l. 8, ccxxiii. Weston, E. R. Yorkshire.

No. 24.—Skull, old *ad.* Ossicle of lambdoid 37 × 25 mm.: faint post-coronal depression: small pterygo-spinous ridge. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 26.—Skull, old *ad.* Right malar wanting. Teeth lost. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 27.—Skull, *ad.* Both zygomata, the right temporal, and part of the occipital wanting. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 28.—Skull, *ad.* Left temporal wanting, occipital and left parietal injured. Posterior sagittal and lambdoid ossicles. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 30.—Cranium, old *ad.* Occipital condyles wanting. Sagittal groove. Thurnam, *loc. cit. sub* No. 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 31.—Calvaria, *ad.* Thurnam, *loc. cit. sub* 21. Dinnington, near Rotherham, W. R. Yorkshire.

No. 39.—Skull with mandible, *ad.* Right infra-orbital suture on face: perforation in anterior wall of left external auditory meatus. *Crania britannica*, Vol. I. pp. 244, 245, cvii. Rodmarton, Gloucestershire.

No. 43.—Calvaria with temporals, detached maxillae, and mandible, young *ad.* A tongue, 20 × 10 mm., projecting from supra-occipital into right parietal, in the middle of the right lambdoid (a fused Wormian?). Rolleston MS., "Barrows not as yet described." Birdlip, Gloucestershire.

No. 46.—Skull with mandible, *ad.* Very imperfect. Rolleston, *Journ. Anthrop. Inst.* v. p. 149 (1875). Swell, Gloucestershire.

No. 63.—Skull with mandible, old *ad.* Very fragmentary. B. B. p. 528, l. 13. Upper Swell, Gloucestershire.

No. 75.—Skull with mandible, *ad.* Both skull and mandible injured. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 78.—Skull with mandible, *ad.* Much broken. Mandible without right condyle. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 79.—Cranium, young *ad.* Left temporal, frontal and right parietal broken. Ossicle of left pterion, 20 × 12 mm. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

No. 85.—Skull with mandible, young *ad.* Broken: defect on left side involving parietal occipital and temporal; condyles gone. *m* 3 coming into place. Bilateral perforation in anterior wall of external auditory meatus. Ackerman and Thurnam, *loc. cit. sub* No. 77. Crawley, near Witney, Oxfordshire.

ROUND BARROWS (MALE).

No. 90.—Cranium with mandible, old *ad.* Basis cranii much broken, left condyle of mandible wanting. Maxillae and fragments in box. Post-parietal flattening with slight sagittal groove: torus occipitalis. B. B. p. 382, clxvii. Ashfell, Kirkby Stephen, Westmoreland. G. C.

No. 93.—Skull, *ad.* Sphenoid, ethmoid, left temporal and left malar wanting. Large healed wound in right frontal; slight sagittal ridge; torus occipitalis; imperfect porus crotaphitico-buccinatorius. B. B. p. 136, l. 41, ii. figured, p. 602. Langton Wold, E. R. Yorkshire. G. C.

No. 96.—Skull with mandible, young *ad.* Left temporal and parietal broken. Signs of injury to left frontal. B. B. p. 141, v. figured on p. 578. Heslerton Wold, E. R. Yorkshire. G. C.

No. 102.—Calvaria, old *ad.* B. B. p. 147, ix. Sherburn Wold, E. R. Yorkshire. G. C.

No. 111.—Skull with mandible, young *ad.* Ethmoid, sphenoid and right malar wanting: fragments in a box. B. B. p. 163, l. 25. Ganton, E. R. Yorkshire. G. C.

No. 114.—Calvaria with temporals, and mandible, *ad.* B. B. p. 166, l. 34, xxii. Ganton, E. R. Yorkshire. G. C.

No. 117.—Cranium with mandible, young *ad.* Malars *in situ*; sphenoid wanting. Ossicles of lambdoid. B. B. p. 174, l. 32, xxvii. Ganton, E. R. Yorkshire. G. C.

No. 118.—Skull with mandible, *ad.* Basis cranii wanting, the rest much broken. Metopic suture. B. B. p. 176, l. 7, xxviii. Ganton, E. R. Yorkshire. G. C.

No. 119.—Calvaria, with portion of left temporal and malars, mandible, *ad.* B. B. p. 176, l. 26, xxviii. Ganton, E. R. Yorkshire. G. C.

No. 122.—Calvaria with temporals and imperfect mandible, old *ad.* Ossicles of lambdoid partly fused; faint torus occipitalis, narrow auditory meatus. B. B. p. 183, l. 11 from bottom, xxxiv. Willerby, E. R. Yorkshire. G. C.

No. 123.—Skull with mandible, young *ad.* Left temporal, pterygoid, and most of occipital wanting; right squamosal wanting. Slight torus occipitalis. B. B. p. 183, l. 5–16, xxxiv. Willerby, E. R. Yorkshire. G. C.

No. 125.—Skull with mandible, *ad.* Basis cranii and right temporal wanting, mandible without left condyle. B. B. p. 191, last line, xli. figured p. 616. Helperthorpe, E. R. Yorkshire. G. C.

No. 126.—Calvaria with portions of temporals, *ad.* B. B. p. 193, xlii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 127.—Skull with mandible, *ad.* Somewhat distorted; sphenoid and ethmoid wanting. Metopic suture; infra-orbital suture on face. B. B. p. 193, l. 14, xlii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 132.—Calvaria with left malar and mandible, *ad.* Lower jaw without right coronoid process. Fragments of other bones in box. Two ossicles in lambdoid, 85 × 15 mm. and 20 × 16 mm. B. B. p. 195, l. 18, xliii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 133.—Cranium, *ad.* Left occipital condyle wanting. The rest much broken. Box with loose fragments. Torus occipitalis. B. B. p. 195, l. 33, xliii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 134.—Calvaria with temporals and mandible, *ad.* Ascending portion of right ramus of mandible broken. Slight torus occipitalis. B. B. p. 198, l. 11, xliv. figured p. 619.

No. 135.—Skull with mandible, young *ad.* Basis cranii wanting. B. B. p. 200, l. 19, xlvi. figured ? p. 570. Weaverthorpe, E. R. Yorkshire. G. C.

No. 137.—Calvaria, loose maxillae, malars, etc., and mandible, *ad.* B. B. p. 201, l. 6, xlvi. Weaverthorpe, E. R. Yorkshire. G. C.

No. 138.—Skull with mandible, *ad.* Ethmoid gone. Parietals much broken: mandible without left condyle. B. B. p. 206, l. 11, xlix. Helperthorpe, E. R. Yorkshire. G. C.

No. 141.—Skull, old *ad.* Basis cranii and face much broken. Ossicles in lambdoid. Torus occipitalis. B. B. p. 207, l. 7, xlix. Helperthorpe, E. R. Yorkshire. G. C.

No. 149.—Skull, young *ad.* The right half of the sphenoid, the ethmoid, and the right maxilla wanting. B. B. p. 217, l. 10, lvii. Cowlam, E. R. Yorkshire. G. C.

No. 153.—Skull with mandible, *ad.* Much broken. Ossicles in lambdoid. B. B. p. 226, l. 8, lix. figured p. 586. Cowlam, E. R. Yorkshire. G. C.

No. 155.—Calvaria with mandible and fragments, *ad.* Slight torus occipitalis; *post mortem* compression. B. B. p. 230, l. 17, lxi. Rudstone, E. R. Yorkshire. G. C.

No. 156.—Skull with mandible, *ad.* Ethmoid, squamosals and occipitals broken; left malar almost entirely wanting. Metopic suture; a few ossicles in the lambdoid; flattening of obelion. B. B. p. 231, l. 1, lxi. Rudstone, E. R. Yorkshire. G. C.

No. 165.—Skull with mandible, *ad.* Left malar and ethmoid wanting. B. B. p. 248, l. 3 from bottom, lxiii. figured p. 590. Rudstone, E. R. Yorkshire. G. C.

No. 169.—Skull with mandible, *ad.* Basis cranii, ethmoid, and right squamosal wanting. Perforation in anterior wall of right external auditory meatus. B. B. p. 265, l. 24, lxviii. Rudstone, E. R. Yorkshire. G. C.

No. 173.—Cranium, with nasals and nasal portion of right maxilla, *ad.* Slight torus occipitalis. B. B. pp. 271, 272, l. 1. Flixton Wold, E. R. Yorkshire. G. C.

No. 178.—Skull with mandible, young *ad.* Basis cranii mostly wanting, right petrosal and squamosal wanting. Ossicle in lambdoid 35 × 17 mm. B. B. p. 274, l. 12, lxx. Flixton Wold, E. R. Yorkshire. G. C.

No. 185.—Skull with mandible, *ad.* (young ?). Sphenoid wanting. Metopic suture. B. B. p. 300, xc. Goodmanham, E. R. Yorkshire. G. C.

No. 186.—Skull with mandible, *ad.* Plagiocephalic; right frontal flattened, right parietal protuberant; bilateral post-coronal depression (distortion partly *post mortem*). Goodmanham, E. R. Yorkshire (?). G. C.

No. 187.—Skull with mandible, young *ad.* Sphenoid, ethmoid and much of both malars broken; mandibular condyles broken. Marked plagiocephaly (*post mortem*?). B. B. p. 301, l. 9, xcii. Goodmanham, E. R. Yorkshire. G. C.

No. 188.—Calvaria with portions of malars and temporals attached; maxillae loose, *ad.* Mandible without angle on right side, without condyle on left. B. B. p. 303, l. 3, xciv. Goodmanham, E. R. Yorkshire. G. C.

No. 190.—Skull with mandible, *ad.* Ethmoidal and presphenoidal regions deficient. Slight sagittal ridge; prominent inion. B. B. p. 308, l. 21, xcix. Goodmanham, E. R. Yorkshire. G. C.

No. 191.—Skull with mandible, *ad.* Left side of head and face largely deficient; mandible without ascending portion of left ramus; condyle and coronary process wanting on right side. Inion pronounced; ossicle of right pterion 30 × 5 mm., between alisphenoid, parietal and frontal. B. B. p. 312, l. 3, ci. Goodmanham, E. R. Yorkshire. G. C.

No. 193.—Skull with mandible, *ad.* Sphenoid and ethmo-nasal regions deficient. Ossicles in right lambdoid. B. B. p. 313, l. 19, ciii. Goodmanham, E. R. Yorkshire. G. C.

No. 195.—Skull with mandible, *ad.* Presphenoidal region deficient. Ossicle in lambdoid 40 × 25 mm. B. B. p. 315, l. 9 from bottom, cv. Goodmanham, E. R. Yorkshire. G. C.

No. 196.—Skull with mandible, young *ad.* Very defective on right side. B. B. p. 318, l. 11 from bottom, cx. Goodmanham, E. R. Yorkshire. G. C.

No. 197.—Skull with mandible, young *ad.* Right side of cranium largely deficient; right ascending ramus of mandible wanting. B. B. p. 318, l. 6 from bottom, cx. Goodmanham, E. R. Yorkshire. G. C.

No. 207.—Skull with mandible, old *ad.* B. B. p. 323, l. 9, cxiii. Goodmanham, E. R. Yorkshire. G. C.

No. 214.—Skull with mandible, *ad.* Base of skull very defective. Mandible without the right condyle. Plagiocephalic. B. B. p. 329, cxx. Goodmanham, E. R. Yorkshire. G. C.

No. 215.—Calvaria with part of left temporal and fragments of mandible, *ad.* (young?). B. B. p. 330, l. 11, cxxi. Goodmanham, E. R. Yorkshire. G. C.

No. 217.—Skull, *ad.* The greater part of the base and all the left temporal wanting. B. B. p. 330, l. 27, cxxi. Goodmanham, E. R. Yorkshire. G. C.

No. 218.—Skull with mandible, *ad.* (young?). Left occipital condyle and sphenethmoid region injured. Mandible with angle broken away on right side. B. B. p. 332, l. 4, cxxiii. Londesborough, E. R. Yorkshire. G. C.

No. 222.—Skull with mandible, old *ad.* Basis cranii and left temporal region absent. Left condyle of mandible broken. Slight torus occipitalis. B. B. p. 447, l. 14 from bottom, ccxvi. Nether Swell, Gloucestershire.

ROUND BARROW (IMMATURE, OR SEX DOUBTFUL).

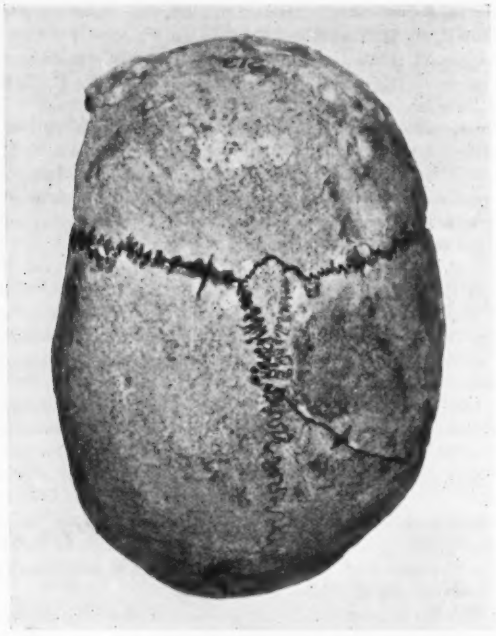
No. 124.—Calvaria with broken petrosals (♂ ?). B. B. p. 185, xxxviii. p. 186, l. 6. Willerby Wold, E. R. Yorkshire. G. C.

No. 140.—Imperfect calvaria (♀ ?) with mandible, post-coronal depression. B. B. p. 206, l. 27, lix. Helperthorpe, E. R. Yorkshire. G. C.

No. 157.—Calvaria, very imperfect (♀ ?) with mandible and box containing maxillae, etc. B. B. p. 232, l. 4, lxi. Rudstone, E. R. Yorkshire. G. C.

No. 181.—Skull with mandible (♀ ?). Right malar, sphenethmoids and nasal region gone. Maxillae separate. Somewhat distorted. B. B. p. 276, l. 23, lxxi. Flixton Wold, E. R. Yorkshire. G. C.

No. 213.—Calvaria, defective. Ossicle of bregma, 26 × 14 mm. (See photograph below.) Group of five ossicles in lambda, the largest about 17 × 20 mm. B. B. p. 328, l. 5, cxviii. Goodmanham, E. R. Yorkshire. G. C.



No. 106.—Skull with mandible, about 6 years. Sphenethmoid region wanting, *m* 1 just coming into place. B. B. p. 161, l. 29, xxi. Potter Brompton Wold, E. R. Yorkshire. G. C.

No. 131.—Skull with mandible, about 6 or 7 years. Base, with sphenoid, ethmoid, nasal and left temporal regions wanting. B. B. p. 195 (query line ?). Weaverthorpe, E. R. Yorkshire. G. C.

No. 139.—Calvaria (young). B. B. p. 206, l. 26, xlix. Helperthorpe, E. R. Yorkshire. G. C.

No. 179.—Skull with mandible (about 12 years). Base imperfect; sphenoid, ethmoid and nasals gone. Ossicle in right lambdoid 15 × 15 mm. B. B. p. 275, l. 7. Flixton Wold, E. R. Yorkshire. G. C.

No. 209.—Skull with mandible (about 8 years). Base and right temporal region wanting. B. B. p. 324, l. 6, cxiv. Goodmanham, E. R. Yorkshire. G. C.

No. 216.—Skull with mandible (about 12 years). Much broken, base wanting. Markedly prognathous. B. B. p. 330, l. 19, cxxi. Goodmanham, E. R. Yorkshire. G. C.

ROUND BARROW (FEMALE).

No. 91.—Calvaria, *ad.* Flattening of obelion. Small ossicle in left lambdoid. Wellburn, N. R. Yorkshire. Cf. Oxford MS. Catalogue.

No. 94.—Skull with mandible, old *ad.* B. B. p. 137, l. 26, ii. Langton Wold, E. R. Yorkshire. G. C.

No. 98.—Cranium, young *ad.* Base largely wanting; otherwise defective. Metopic suture persistent; ossicle in right lambdoid. B. B. p. 146 (line ?), vii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 99.—Skull with mandible, *ad.* Occiput protuberant. B. B. p. 146 (line ?), vii. (figured, p. 608). Sherburn Wold, E. R. Yorkshire. G. C.

No. 100.—Calvaria with right temporal, *ad.* B. B. p. 146 (line ?), vii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 103.—Skull with mandible, *adolescent.* Base, ethmoidal and right temporal regions wanting. Ascending portion of right ramus of mandible wanting. Metopic suture persistent. B. B. p. 150, l. 36, xii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 104.—Skull with mandible, young *ad.* Base defective. Ossicle of lambda 25 × 30 mm.; another ossicle in left lambdoid 20 × 15 mm.; small ossicle of left pterion. B. B. p. 152, l. 23, xiii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 105.—Skull with mandible, young *ad.* Basis cranii imperfect, left malar wanting. Part of occipital, with one condyle, loose in box. Triple ossicle of lambdoid. B. B. p. 154, l. 1, xiii. Sherburn Wold, E. R. Yorkshire. G. C.

No. 110.—Skull without basi-occipital, young *ad.* B. B. p. 163, l. 17, xxi. Ganton, E. R. Yorkshire. G. C.

No. 112.—Skull with mandible, young *ad.* Right side of face and right squamosal and part of frontal wanting. B. B. p. 163, l. 34, xxi. Ganton, E. R. Yorkshire. G. C.

No. 113.—Calvaria, *ad.* Part of base and left temporal attached. Metopic suture persistent. B. B. pp. 166, 167, xxii. Ganton, E. R. Yorkshire. G. C.

No. 115.—Cranium, *ad.* Much broken. Box containing loose parts of face and broken mandible. B. B. p. 166, l. 38, xxii. Ganton, E. R. Yorkshire. G. C.

No. 121.—Cranium with mandible, *ad.* Mandible much broken. Imperfect facial bones in box. Metopic suture persistent. B. B. p. 183, l. 7, xxxiii. Willerby Wold, E. R. Yorkshire. G. C.

No. 130.—Cranium with malar bones and mandible, old *ad.* B. B. p. 195, l. 3, xliii. Weaverthorpe, E. R. Yorkshire. G. C.

No. 136.—Imperfect cranium with broken mandible, young *ad.* Bathrocephalic; ossicles in lambdoid. B. B. p. 200, l. 39, xlvi. Weaverthorpe, E. R. Yorkshire. G. C.

No. 142.—Skull with mandible, *ad.* Base and ethmoidal regions imperfect; mandible without condyles; box with fragments. Metopic suture persistent; numerous ossicles in left lambdoid. B. B. p. 210, l. 4, li. Cowlam, E. R. Yorkshire. G. C.

No. 143.—Skull with mandible, *ad.* Base, sphenoid, ethmoid, left malar, squamosal and parts of frontal and parietal gone. Marked alveolar prognathism. B. B. p. 211, l. 8, lii. Cowlam, E. R. Yorkshire. G. C.

No. 144.—Skull with mandible, *ad.* Sphenoid and ethmoid regions wanting. B. B. p. 211, l. 21, liii. Cowlam, E. R. Yorkshire. G. C.

No. 148.—Cranium with mandible, young *ad.* Most of base gone. Right condyle and coronary process of mandible broken. Box containing maxillae. Ossicles in lambdoid; synostosis of atlas and occipital. B. B. p. 215, l. 37, lvii. Cowlam, E. R. Yorkshire. G. C.

No. 150.—Cranium with mandible, *ad.* Mandible without ascending portion of right ramus. Occipital slightly protuberant. B. B. p. 218, l. 27, lvii. Cowlam, E. R. Yorkshire. G. C.

No. 151.—Cranium with mandible and box of fragments, *ad.* Mandible without right condyle. B. B. p. 219, l. 3, lvii. Cowlam, E. R. Yorkshire. G. C.

No. 159.—Cranium with mandible, *ad.* (old ?). Very defective; mandible without most of left ramus. Box of fragments. Perforation in anterior end of right external auditory meatus. B. B. p. 236, l. 22, lxii. Rudstone, E. R. Yorkshire. G. C.

No. 160.—Skull with mandible, young *ad.* Ossicle of lambdoid 30 × 20 mm. Left imperfect porus crotaphitico-buccinatorius. B. B. p. 237, l. 19, lxii. Rudstone, E. R. Yorkshire. G. C.

No. 162.—Skull with mandible, *ad.* Base, sphenoid and right maxilla wanting. Box con-

taining fragments. Slight post-parietal groove; ossicles in lambdoid; slight bathrocephaly. B. B. p. 248, l. 12, lxiii. Rudstone, E. R. Yorkshire. G. C.

No. 166.—Calvaria, petrosals attached, with mandible, *ad.* B. B. p. 250, l. 19, lxiii. Rudstone, E. R. Yorkshire. G. C.

No. 171.—Skull with mandible, *ad.* Base entirely gone, together with sphenoid and ethmoid. Faint torus occipitalis. B. B. p. 556, l. 7, ccxxxiv. Rudstone, E. R. Yorkshire. G. C.

No. 177.—Skull with mandible, *ad.* Whole basis and right malar wanting. B. B. p. 273, l. 14, lxx. Flixton Wold, E. R. Yorkshire. G. C.

No. 180.—Skull with mandible, young *ad.* Ossicle of left pterion 15 × 8 mm. between alisphenoid and frontal. Ossicle of left asterion 10 × 12 mm. Tongue 15 × 8 mm. from occipital into left parietal. B. B. p. 275, l. 33, lxxi. Flixton Wold, E. R. Yorkshire. G. C.

No. 192.—Skull with mandible, *ad.* Basi-occipital and sphenoidal regions wanting. Large ossicles in left lambdoid nearly fused. B. B. p. 313, l. 3, ciii. Goodmanham, E. R. Yorkshire. G. C.

No. 199.—Skull with mandible, young *ad.* Basi-cranial axis wanting; mandible without left condyle. Ossicle of lambdoid 37 × 33 mm. B. B. p. 319, l. 17, ci. Goodmanham, E. R. Yorkshire. G. C.

No. 200.—Skull with mandible, *ad.* (young?). Basi-cranial axis and left malar wanting. Left angle of mandible wanting. Traces of torus occipitalis. B. B. p. 319, l. 36, ci. Goodmanham, E. R. Yorkshire. G. C.

No. 201.—Calvaria with temporals and malars attached, and with mandible, *ad.* B. B. p. 230, l. 12, ci. Goodmanham, E. R. Yorkshire. G. C.

No. 202.—Skull with mandible, *ad.* Right malar, temporal and much of frontal and parietal wanting. Mandible without condyle or angle on right side. Slight posterior sagittal groove. B. B. p. 321, l. 14, cxii. Goodmanham, E. R. Yorkshire. G. C.

No. 204.—Imperfect cranium with mandible, *ad.* Metopic suture persistent. Ossicle 23 × 15 about 20 mm. behind bregma. Faint post-coronal depression. B. B. p. 322, l. 5, cxiii. Goodmanham, E. R. Yorkshire. G. C.

No. 206.—Calvaria with left temporal, *ad.* Slight flattening of obelion. B. B. p. 323, l. 4, cxiii. Goodmanham, E. R. Yorkshire. G. C.

No. 211.—Cranium with loose maxillae and mandible, *ad.* (young?). Basis cranii wanting. Mandible broken at symphysis and otherwise imperfect. Faint torus occipitalis; double ossicle of lambdoid about 40 × 20 mm. B. B. p. 326, l. 33, cxvii. Goodmanham, E. R. Yorkshire. G. C.

No. 212.—Skull with mandible, young *ad.* Basis and sphenoid imperfect. Metopic suture persistent. Ossicles in lambdoid. B. B. p. 327, l. 15, cxvii. Goodmanham, E. R. Yorkshire. G. C.

No. 221.—Skull, *ad.* (old?). Sphenoid, ethmoid and left malar defective. Post-coronal depression. Ossicles of lambda and in lambdoid suture. Slight bathrocephaly. Gatcombe Park, Minchinhampton, Gloucestershire.

THE EGG OF CUCULUS CANORUS.

AN ATTEMPT TO ASCERTAIN FROM THE DIMENSIONS OF THE CUCKOO'S EGG IF THE SPECIES IS TENDING TO BREAK UP INTO SUB-SPECIES, EACH EXHIBITING A PREFERENCE FOR SOME ONE FOSTER-PARENT.

SECOND MEMOIR.

By OSWALD H. LATTER.

(1) *On the Existence of Cuckoo Gentes.*

In my previous paper (*Biometrika*, Vol. I. No. 2, January 1902, pp. 164—176) I examined this and certain other questions relating to the egg of the Cuckoo. On that occasion I had measurements of only 243 eggs. I have now been able to obtain the dimensions of 1572 specimens (including those dealt with previously) and am therefore in a position to speak with more confidence. Of this number, 717 eggs were measured by myself; and I must here take the opportunity of recording my grateful thanks to Prof. A. Newton of Cambridge, Dr E. Hartert of the Tring Museum, Mr F. Norgate of Bury S. Edmunds, Mr J. H. Gurney of Keswick Hall, Norwich, and many other private collectors for their courtesy in granting me help and access to their collections. I am further indebted to a large number of my pupils at Charterhouse for assistance in tabulating the measurements and in checking the accuracy of the calculations. The remaining 855 specimens were measured by Eugene Rey of Leipzig, and the dimensions published in *Zoologische Vorträge herausgegeben von W. Marshall*, Leipzig, 11th Heft, 1892; *Ornithologische Monatschrift d. Deutschen Vereins z. Schutze der Vogelwelt*, XIX. Jahrgang, 1894, No. 5; and *Journal für Ornithologie*, XLIII. Jan. 1895.

In the present instance I have confined myself to the attempt to ascertain if the eggs of Cuckoos deposited in the nests of any one species stand out as a set apart from Cuckoo's eggs deposited elsewhere. The series includes eggs taken from the nests of no less than eighty-four different species of foster-parents, in addition to a number whose history was not ascertainable. From the nests of sixteen species of foster-parents sufficient Cuckoo's eggs were obtained to make it

worth while to test the point in question in respect of the dimensions of length and breadth. The method employed is fully explained in my previous paper, but it will be convenient to re-state it. The mean (M) length or breadth, as the case may be, of all the Cuckoo's eggs is first computed, thence the standard deviation (σ) is obtained by the formula $\sigma^2 = \frac{\text{sum}\,(M-x)^2}{n}$, where x = the measurement of any one egg and n = the number of eggs measured: the coefficient of variation, $\frac{100\,\sigma}{M}$, can then be found. To test whether the deviation occurring in Cuckoo's eggs deposited in the nests of any particular species of foster-parent is significant, M_r is taken as the mean of the whole race of Cuckoos, and M_s as the mean of Cuckoo's eggs found in the nests of the foster-parent species: the standard deviation (σ_s) of such eggs is also ascertained. The value of $M_r - M_s$ is then compared with that of $0{\cdot}67449\sqrt{\frac{\sigma_r^2}{n_1}+\frac{\sigma_s^2}{n_2}}$ (where n_1 = total number of Cuckoo's eggs, and n_2 = the number of Cuckoo's eggs in the nests of the species in question), which is the probable error of $M_r - M_s$ due to random sampling. If the value of $M_r - M_s$ be not at least 2 to 3 times as great as the value of the other expression, then the difference of M_r and M_s is not definitely significant. It will be seen by reference to the tabulated summary below that in the case of four of the sixteen sets examined, viz. Robin-Cuckoos, Wren-Cuckoos, Whitethroat-Cuckoos, and Hedge-Sparrow-Cuckoos, the differences are significant in respect of both length and breadth. Of the remainder, five, viz. Garden-Warbler-Cuckoos, Barred-Warbler-Cuckoos, Marsh-Warbler-Cuckoos, Reed-Warbler-Cuckoos, and Tree-Pipit-Cuckoos, exhibit differences significant in one dimension, but doubtful in the other; two, viz. Sedge-Warbler-Cuckoos and White-Wagtail-Cuckoos, exhibit significant differences in one dimension only; one, viz. Meadow-Pipit-Cuckoos, exhibit doubtfully significant differences in but one dimension; and four, viz. Redstart-Cuckoos, Red-backed-Shrike-Cuckoos, Pied-Wagtail-Cuckoos, and Yellow-Ammer-Cuckoos, do not present differences significant in either dimension. The balance of evidence is thus decidedly in favour of there being distinct sets of Cuckoos. As a result it now seems fairly certain that Prof. A. Newton's suggestion (*Dictionary of Birds*, p. 123) is correct, and that there are certain "gentes" of Cuckoos whose members being closely related lay eggs of somewhat similar dimensions, and in the main confine their attentions, generation after generation, each to its own particular variety of foster-parent. In other words, the evidence here adduced points to the conclusion that the species *Cuculus canorus* is tending to break up into a number of sub-species, each with its own particular habits in respect of selecting a foster-parent. There are recorded numerous instances of individual Cuckoos adhering to one species of foster-parent whenever possible, but it appears that in the event of failure to find a nest of the species preferred the Cuckoo places her egg in any nest that comes handy. This fact probably accounts for the very large number of species in whose nests Cuckoo's eggs have been found, and emphasises the importance of obtaining large numbers of measurements for investigations of this character.

TABLE I.

Summary of Results.

Group	Length in Millimetres					Breadth in Millimetres				Remarks
	Number of Eggs	Mean (M)	Standard Deviation (σ)	Coefficient of Variation	Significance Test*	Mean (M)	Standard Deviation (σ)	Coefficient of Variation	Significance Test*	
Cuckoo (Race)	1572	22·3	·9642	4·32	—	16·5	·5952	3·60	—	— —
Robin-Cuckoo	78	22·8	1·1638	5·105	5·5	16·8	·7127	4·242	6	L. and B. significant
Garden-Warbler-Cuckoo ...	91	21·9	·7860	3·587	7·01	16·4	·4900	2·980	2·8	L. significant and B. probably so
Meadow-Pipit-Cuckoo ...	86	22·3	·8851	3·969	—	16·6	·5238	3·149	2·57	B. probably significant
Wren-Cuckoo	54	21·4	·9375	4·380	10	16·2	·6578	4·060	5·0	L. and B. significant
Redstart-Cuckoo	64	22·4	·8840	3·857	1·3	16·6	·5225	3·16	—	Neither significant
Sedge-Warbler-Cuckoo ...	48	22·1	·8642	3·91	11·1	16·4	·6121	3·730	1·6	L. only significant
Barred-Warbler-Cuckoo ...	35	21·8	·8617	3·952	5·0	16·3	·6090	3·736	2·85	L. significant and B. probably so
Whitethroat-Cuckoo ...	37	21·8	·8545	3·919	5·2	16·2	·6071	3·747	4·4	L. and B. significant
Marsh-Warbler-Cuckoo ...	37	22·1	·8330	3·769	2·1	16·3	·4647	2·850	4·0	L. perhaps : B. significant
Red-backed-Shrike-Cuckoo	307	22·3	·7976	3·58	—	16·5	·4909	2·975	—	Neither significant
White-Wagtail-Cuckoo ...	115	22·4	·7606	3·398	1·6	16·7	·6766	4·051	4·6	B. probably significant
Pied-Wagtail-Cuckoo ...	85	22·3	·9192	4·122	—	16·5	·5242	3·176	—	Neither significant
Reed-Warbler-Cuckoo ...	66	22·6	1·0219	4·51	3·2	16·6	·5768	3·470	2·04	L. significant and B. perhaps so
Tree-Pipit-Cuckoo ...	48	22·6	·9514	4·209	3·0	16·6	·4394	2·647	2·3	L. significant and B. perhaps so
Hedge-Sparrow-Cuckoo ...	58	22·6	·8759	3·875	3·75	16·8	·4970	2·958	6·6	L. and B. significant
Yellow-Ammer-Cuckoo ...	31	22·2	·9846	4·43	0·8	16·5	·4981	3·0	—	Neither significant

4 species differ significantly in both dimensions.
5 „ „ one and perhaps in both.
2 „ „ one only.
1 „ „ perhaps one only.
4 „ „ neither dimension.

* Ratio of difference of racial and group means to the probable error of this difference.

We note that judged by the coefficient of variation the Cuckoo's egg is more variable than the eggs for the sub-groups for

Length in 12 out of 16 cases,

Breadth in 10 out of 16 cases.

It is less variable in both breadth and length in the cases of Robin-Cuckoo and Wren-Cuckoo, less variable in length only in Yellow-Ammer-Cuckoo and Reed-Warbler-Cuckoo, and less variable in breadth only in Sedge-Warbler-Cuckoo, Barred-Warbler-Cuckoo, Whitethroat-Cuckoo, and White-Wagtail-Cuckoo. We therefore see that as a rule the egg of the *gens* is less variable than that of the race.

(2) *On the Relation of the Gens Egg to the Foster-Mother's Egg.*

Taking these 16 cases and correlating the dimensions of the Cuckoo's egg with those of the foster-mother, we find:

Correlation of length of Cuckoo's and foster-mother's egg* = ·10,

Correlation of breadth of Cuckoo's and foster-mother's egg = ·24.

These are not very large values, and the number of cases dealt with is only 16, but it does appear that there is some relationship between the size

TABLE II.

Bulk-Modulus of Cuckoo's and Foster-Mother's Eggs.

Species	Size of Foster-Mother's Egg	Size of Frequenting Cuckoo's Egg
Robin	4791	6435
Garden-Warbler ...	5197	5890
Meadow-Pipit ...	4142	6145
Wren	2855	5616
Redstart	3724	6173
Sedge-Warbler ...	3234	5944
Barred-Warbler ...	5478	5792
Whitethroat ...	3831	5721
Marsh-Warbler ...	3724	5872
Red-backed-Shrike	6359	6071
White-Wagtail ...	4877	6247
Pied-Wagtail ...	4690	6071
Reed-Warbler ...	2871	6228
Tree-Pipit ...	4560	6228
Hedge-Sparrow ...	4322	6379
Yellow-Ammer ...	5504	6044
Mean	4385 mm.^{3}	6053·5 mm.^{3}
Standard Deviation	959·13 mm.^{3}	225·72 mm.^{3}

* Mean Length of Cuckoo's Egg, 22·225; of Foster-Parent's Egg, 19·806; the mean breadths are respectively 16·500 and 14·763; the corresponding standard deviations are: Length ·3544 and 1·6414; Breadth ·1837 and 1·0971.

of the foster-parents' eggs and those which the Cuckoo lays in their nest. It is worth while considering this relationship from another standpoint. The bulk of an egg must be approximately proportional to its length × (breadth)^2, and we may take this roughly as a bulk-modulus. Using it we obtain the foregoing Table II. for the bulk-moduli in cubic millimetres. This leads to a correlation of ·1746, or ·17, say, and confirms the previous results. There thus appears to be really a small relation between the bulk of the Cuckoo's egg and that of the foster-parent. The results are based on only 16 classes, which is, of course, small for determining a correlation, but it would seem that we must, since all three measurements give a positive correlation, assume some slight relationship between size of Cuckoo's egg and that of the foster-parent. This relation might be clearer and less irregular, if we had measurements of fairly long series of the eggs of the foster-parents. It is perhaps worth noting that the breadth measurement is the one in which the resemblance is closer. We may therefore ask: Is it possible that the differentiation in size of egg, which we find as we pass from the foster-parent of one to the foster-parent of a second species, is merely due to a slight constriction of the oviduct, instinctive or otherwise, following on the choice of a foster-parent? Were this the case, it would be unnecessary to suppose separate Cuckoo *gentes*. Now the relation between length and breadth of egg has been worked out for Thrush, Blackbird, House-Sparrow and Linnet and the value of the correlation lies between ·35 and ·40. We should expect therefore that if the differentiation of the Cuckoo's egg were produced solely by such a constriction of the oviduct, that the length of the egg would be increased, or the above correlation between length and breadth be at any rate reduced. We find, however, that if the length and breadth of the 16 Cuckoo groups be correlated, the resulting value is ·91. There can hardly be a doubt therefore that the variation in the Cuckoo's egg as we pass from one species of foster-parent to a second is a real variation in bulk, the length and breadth changing in a closely correlated manner. Thus although the egg is only slightly correlated with the size of the foster-parent's egg it varies as a whole in a manner not consonant with a mere constriction of the oviduct, and the fact that the breadths of foster-mother's and Cuckoo's eggs are more closely correlated than their lengths, does not enable us to dispense with a theory of *gentes*.

Table IV. shows the dimensions of the eggs of the foster-parents for comparison with those of the Cuckoo frequenting their nests. The approximate sizes of the birds are also inserted.

(3) *On the Distribution of Frequency in the Cuckoo's Egg.*

A consideration of the first two columns of totals under the heading "Race" in Table III., shows that Rey has measured the great bulk of his eggs to only ·5 mm. accuracy; he has probably not used a vernier and the result is that his frequencies are crowded up on the whole and half millimetres; a similar crowding although less conspicuous is obvious in the additional 717 measurements. This unconscious

TABLE III. *Measurements of Cuckoo's Eggs.*

Length of Egg.

Length in mm.	Rey	Race: Latter	Race: Total	Gentes: Robin	Garden-Warbler	Meadow-Pipit	Wren	Redstart	Sedge-Warbler	Barred-Warbler	White-throat	Marsh-Warbler	Red-backed-Shrike	White-Wagtail	Pied-Wagtail	Reed-Warbler	Tree-Pipit	Hedge-Sparrow	Yellow-Ammer
19·1	—	1	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19·6	—	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—
19·7	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19·8	—	1	1	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	1
19·9	—	2	2	—	—	—	—	2	—	—	—	—	1	—	1	1	—	—	—
20·0	13	3	16	1	2	—	6	—	—	—	—	—	—	—	—	—	—	—	—
20·1	2	4	6	—	—	1	—	1	2	1	—	1	1	—	—	3	2	—	3
20·2	4	4	8	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
20·3	1	1	2	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	1
20·4	1	1	2	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—
20·5	15	2	17	—	3	—	1	—	—	4	1	—	4	1	1	3	2	2	1
20·6	6	2	8	1	—	1	1	—	—	—	—	—	2	1	—	—	—	—	—
20·7	8	2	10	—	1	1	1	—	—	—	1	—	3	—	—	—	—	—	—
20·8	7	6	13	—	2	—	3	—	—	1	—	1	1	1	1	1	—	—	—
20·9	4	15	19	1	—	2	3	—	1	—	1	—	1	2	2	—	2	—	—
21·0	56	18	74	5	7	1	7	4	3	3	5	1	13	4	2	3	2	1	1
21·1	6	13	19	2	2	1	—	—	—	—	—	—	1	1	—	—	—	3	1
21·2	14	17	31	—	4	—	1	1	—	—	—	1	4	3	—	3	—	1	1
21·3	12	13	25	1	1	1	4	—	2	—	1	—	4	2	1	1	—	—	—
21·4	9	9	18	—	3	1	—	1	—	2	1	2	1	—	2	1	—	—	1
21·5	46	8	54	1	8	2	3	2	4	2	1	—	9	4	4	4	2	2	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
21·6	11	8	19	2	2	2	1	1	3	2	1	1	—	—	—	—	—	—	—
21·7	20	16	36	1	1	4	—	2	2	—	2	1	10	1	2	2	—	3	—
21·8	17	24	41	2	2	2	—	1	2	—	1	—	10	3	5	1	2	—	—
21·9	17	39	56	1	1	8	—	1	2	2	2	4	9	18	6	5	1	9	2
22·0	131	58	189	—	2	10	6	16	6	7	3	4	48	5	5	1	6	—	2

22·1	18	29	47	2	2	2	3	1	2	—	1	1	10	2	1	2	1	5	—
22·2	30	29	59	1	3	5	—	2	3	2	1	1	13	5	3	3	1	—	1
22·3	34	33	67	1	7	6	1	2	1	1	3	3	18	4	4	2	—	2	1
22·4	16	21	37	4	2	2	2	—	2	—	2	—	8	5	1	—	2	—	1
22·5	63	29	92	3	12	5	3	4	2	6	1	1	18	9	4	4	3	7	2
22·6	19	19	38	3	2	5	—	—	—	—	—	—	7	7	2	—	—	1	1
22·7	35	19	54	2	2	1	—	2	—	—	2	2	16	8	2	1	2	3	2
22·8	26	33	59	2	—	4	2	1	2	—	—	1	12	1	9	3	—	4	—
22·9	14	29	43	3	4	2	—	—	1	—	2	2	6	4	2	1	2	—	1
23·0	74	39	113	11	3	2	—	6	2	2	—	2	31	14	4	6	1	5	2
23·1	14	24	38	4	1	—	1	1	—	—	—	2	5	3	4	2	1	3	—
23·2	10	23	33	—	2	—	—	3	1	1	—	—	6	1	3	1	2	1	2
23·3	10	28	38	1	—	4	—	1	1	—	—	1	7	3	1	—	4	1	2
23·4	1	5	6	—	—	1	—	—	1	—	—	—	1	—	1	—	1	—	—
23·5	22	8	30	2	—	1	2	4	1	—	—	—	7	1	1	6	1	1	1
23·6	4	7	11	2	—	—	—	—	—	—	—	—	3	—	2	2	1	—	—
23·7	10	5	15	1	—	1	—	1	—	—	—	—	5	2	—	1	—	1	—
23·8	1	14	15	1	1	1	—	2	—	—	—	—	1	—	1	—	3	2	—
23·9	3	14	17	3	—	2	—	—	1	—	—	—	1	—	1	—	—	1	—
24·0	27	17	44	7	—	1	—	3	—	1	1	1	6	4	3	4	4	2	—
24·1	1	1	2	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—
24·2	1	7	8	—	—	1	—	—	—	—	—	—	—	—	—	—	2	—	—
24·3	3	1	4	1	—	1	—	—	—	—	—	—	2	—	—	—	—	—	—
24·4	—	3	3	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—
24·5	7	1	8	—	—	—	—	1	1	—	—	—	1	1	—	2	—	—	—
24·6	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
24·7	3	2	5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
24·8	—	3	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
24·9	—	1	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—
25·0	6	1	7	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—
25·1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
25·4	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
25·5	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
26·5	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	855	717	1572	78	91	86	54	64	48	35	37	37	307	115	85	66	48	58	31

TABLE III.—(*continued*).

Breadth of Egg.

	Race			Gentes																
Breadth in mm.	Rey	Latter	Total	Robin	Garden-Warbler	Meadow-Pipit	Wren	Redstart	Sedge-Warbler	Barred-Warbler	Whitethroat	Marsh-Warbler	Red-backed-Shrike	White-Wagtail	Pied-Wagtail	Reed-Warbler	Tree-Pipit	Hedge-Sparrow	Yellow-Ammer
14·0	—	1	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·5	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·7	1	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—
14·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·9	—	2	2	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
15·0	9	2	11	—	—	—	2	2	—	2	—	—	2	1	—	—	—	—	—
15·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15·2	3	3	6	1	—	1	1	—	—	—	—	—	—	1	—	—	—	—	—
15·3	5	6	11	—	—	—	2	—	—	—	1	—	1	—	3	—	—	—	—
15·4	2	7	9	—	1	—	—	—	2	—	—	—	—	1	—	—	1	—	—
15·5	22	6	28	1	4	—	5	—	1	2	1	1	2	—	1	1	—	—	1
15·6	6	11	17	1	—	1	—	—	2	1	1	1	—	3	—	—	—	—	—
15·7	27	10	37	—	3	3	4	2	3	2	2	1	7	1	1	3	1	1	—
15·8	16	25	41	1	1	2	1	1	2	1	3	3	5	—	2	—	1	1	2
15·9	11	34	45	2	1	2	3	—	2	2	—	2	6	2	4	1	—	—	3
16·0	136	68	204	10	24	2	10	10	4	4	8	3	45	13	8	10	5	3	3
16·1	15	36	51	1	7	4	1	2	3	1	—	2	5	1	3	5	—	2	2
16·2	38	64	102	3	6	8	4	4	—	2	2	4	19	4	10	3	3	3	2
16·3	29	33	62	3	—	4	—	1	2	1	2	3	18	2	5	1	4	2	2
16·4	22	31	53	3	3	6	2	1	2	—	3	1	12	1	2	1	1	2	1
16·5	124	34	158	7	8	4	3	14	7	6	4	2	44	16	6	7	6	4	2
16·6	23	22	45	2	4	2	1	1	1	—	2	2	10	3	3	2	2	2	—
16·7	45	32	77	2	5	3	1	2	1	2	—	3	20	5	5	2	4	8	2
16·8	54	38	92	3	6	10	3	2	—	1	—	2	35	1	3	—	3	3	1
16·9	18	51	69	4	—	6	—	1	1	1	3	2	6	4	8	3	1	3	—
17·0	128	70	198	7	12	13	4	17	7	4	3	2	34	24	11	11	8	7	7
17·1	17	38	55	4	2	4	2	1	3	1	—	2	11	2	3	3	2	5	—
17·2	17	25	42	2	1	3	1	1	1	1	—	1	8	1	3	3	—	2	2
17·3	15	21	36	4	1	4	1	—	1	—	—	—	4	4	1	3	2	2	1
17·4	6	7	13	3	—	—	1	—	1	—	—	—	1	2	—	—	—	—	—
17·5	29	9	38	5	2	1	1	1	—	1	1	—	8	6	1	5	1	4	—
17·6	6	3	9	—	—	1	—	—	—	—	—	—	—	3	1	—	—	1	—
17·7	12	7	19	2	—	—	1	1	2	—	—	—	1	6	1	1	—	2	—
17·8	5	7	12	2	—	1	—	—	—	—	—	—	—	2	—	—	1	1	—
17·9	4	4	8	—	—	—	—	—	—	—	—	—	—	4	—	1	2	—	—
18·0	9	4	13	3	—	1	—	—	—	—	—	—	2	1	—	—	—	—	—
18·1	—	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18·2	1	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18·8	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19·2	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	855	717	1572	78	91	86	54	64	48	35	37	37	307	115	85	66	48	58	31

Of the remaining 332 Cuckoo's eggs the measurements of which are given in the totals: 46 were not classed, 11 were in Reed-Bunting's nests, 17 in Wood-Warbler's, 17 in Linnet's, 14 in Chaffinch's, 4 in Bullfinch's, 14 in various Lark's, 14 in Yellow-Wagtail's, 15 in Blueheaded-Yellow-Wagtail's, 10 in Spotted-Flycatcher's, 15 in Willow-Wren's, 5 in Nightingale's, 15 in Greenfinch's, 20 in Blackcap's, 7 in Whinchat's, 14 in Lesser-Whitethroat's, 6 in Wheatear's nests, and the remainder in numbers of 5 and under in the nests of various other species.

TABLE IV.

	Cuckoo's Eggs in mm.		Foster-Parent's Egg in mm.		Approximate Length of Bird in inches
	Length	Breadth	Length	Breadth	
Robin	22·8	16·8	20·2	15·4	$5\frac{3}{4}$—6
Garden-Warbler ...	21·9	16·4	*20·3	16·0	$5\frac{1}{4}$
Meadow-Pipit ...	22·3	16·6	19·7	14·5	$5\frac{3}{4}$—6
Wren	21·4	16·2	*17·7	12·7	$3\frac{3}{4}$
Redstart	22·4	16·6	*19·0	14·0	$5\frac{1}{4}$
Sedge-Warbler ...	22·1	16·4	*16·5	14·0	$4\frac{3}{4}$
Barred-Warbler ...	21·8	16·3	*22·8	15·5	$6\frac{1}{4}$
Whitethroat ...	21·8	16·2	*19·0	14·2	$5\frac{1}{4}$
Marsh-Warbler ...	22·1	16·3	*19·0	14·0	$5\frac{1}{4}$
Red-backed-Shrike..	22·3	16·5	*22·8	16·7	$7\frac{1}{4}$
White-Wagtail ...	22·4	16·7	*20·3	15·5	$7\frac{1}{4}$
Pied-Wagtail ...	22·3	16·5	*20·3	15·2	$7\frac{1}{4}$
Reed-Warbler ...	22·6	16·6	*17·8	12·7	$5\frac{1}{2}$
Tree-Pipit	22·6	16·6	20·0	15·1	6 —$6\frac{1}{2}$
Hedge-Sparrow ...	22·6	16·8	20·0	14·7	$5\frac{1}{2}$—6
Yellow-Ammer ...	22·2	16·5	21·5	16·0	$6\frac{1}{2}$—7
Cuckoo	22·3	16·5	—	—	14

TABLE V.

Frequency Distributions of Cuckoo's Eggs.

A. With ·5 mm. unit of grouping.

Length	Number		Breadth	Number	
	Rey + O. H. L.	O. H. L. only		Rey + O. H. L.	O. H. L. only
18·75—19·25	1	1	13·75—14·25	1	1
19·25—19·75	3	2	14·25—14·75	2	1
19·75—20·25	33	14	14·75—15·25	19	7
20·25—20·75	39	8	15·25—15·75	102	40
20·75—21·25	156	69	15·75—16·25	443	227
21·25—21·75	152	54	16·25—16·75	395	152
21·75—22·25	392	179	16·75—17·25	456	222
22·25—22·75	288	121	17·25—17·75	115	47
22·75—23·25	286	148	17·75—18·25	37	18
23·25—23·75	100	53	18·25—18·75	0	0
23·75—24·25	86	53	18·75—19·25	2	2
24·25—24·75	21	8			
24·75—25·25	12	5	Totals	1572	717
25·25—25·75	2	1			
25·75—26·25	0	0			
26·25—26·75	1	1			
Totals	1572	717			

* Computed from measurements given in inches by W. J. Gordon in *Our Country's Birds.*

B. With 1 mm. unit of grouping.

Length	Number			Breadth	Number		
	Rey + O. H. L.	Rey only	O. H. L. only		Rey + O. H. L.	Rey only	O. H. L. only
18·75—19·75	4	1	3	13·75—14·75	3	1	2
19·75—20·75	72	50	22	14·75—15·75	121	74	47
20·75—21·75	308	185	123	15·75—16·75	838	459	379
21·75—22·75	680	380	300	16·75—17·75	571	302	269
22·75—23·75	386	185	201	17·75—18·75	37	19	18
23·75—24·75	107	46	61	18·75—19·75	2	0	2
24·75—25·75	14	8	6				
25·75—26·75	1	0	1	Totals	1572	855	717
Totals	1572	855	717				

prejudice in favour of the half and whole millimetres may have tended to obscure some of the points involved and renders it difficult to deal satisfactorily with the distribution of frequency. The multimodal character of the previous distributions (see *Biometrika*, Vol. I. pp. 175 and 176) was chiefly due to the manner in which this emphasis of certain values upset the grouping. Admitting the defect the only legitimate grouping is that which takes an entire millimetre as its unit, but this is very coarse, especially in the case of the breadths, for the purpose of drawing any conclusions. In Table V. B the frequencies are given with 1 mm. unit of grouping and we see that the ·5 mm. modes have disappeared, but the grouping is very coarse. Working, however, from these we find:

TABLE VI.

Authority	Mean in mm.		Standard Deviation in mm.	
	Length	Breadth	Length	Breadth
Rey	22·266	16·559	·9450	·5955
O. H. L. ...	22·486	16·613	·9546	·6021
Total ...	22·366	16·583	·9557	·5992

An examination of this table seems to indicate that the English Cuckoo's egg is for both dimensions slightly larger and more variable than the German. The difference, however, is not such that we can lay any stress upon it, and in round numbers we may certainly take:

Length of egg 22·4, Breadth 16·6, standard deviation of Length = ·96 and of Breadth = ·60.

The following results were obtained by calculating the normal curves for four sets of frequencies*:

* The determination of P, the goodness of fit, from χ^2 has been made by aid of Palin Elderton's Tables, *Biometrika*, Vol. I. p. 159.

Length				Breadth			
Rey		Latter		Rey		Latter	
Obs.	Calc.	Obs.	Calc.	Obs.	Calc.	Obs.	Calc.
1	3·32	3	1·49	—	—	—	—
50	43·14	22	23·27	1	1·01	2	0·71
185	203·66	123	133·24	74	73·43	47	53·44
380	344·74	300	278·73	459	460·36	379	369·01
185	210·39	201	213·77	302	300·75	269	272·68
46	46·06	61	59·16	19	19·35	18	21·02
8	3·57	6	7·11	0	·10	2	0·14
0	·10	1	0·23	—	—	—	—
$\chi^2=16·36$ $P<·025$		$\chi^2=8·09$ $P=·325$		$\chi^2=·62$ $P>·99$		$\chi^2=28·59$ $P<·00004$	

The odds are thus more than 40 to 1 against Rey's lengths of eggs fitting a normal distribution. But only once in 100 random samples of 855 eggs should we expect a better result than we have got for the breadths, if the distribution be really normal. Turning to the 717 measurements we find the length distribution reasonable on the basis of a normal curve; once in every three random samples of 717 eggs we should get a worse result, but the breadths are quite impossible. This impossibility arises, however, entirely from the two giant breadths of 18·8 and 19·2. They are undoubtedly abnormalities and if they be excluded, the fit is a good one, i.e. about 55 samples in 100 would give a worse result. With regard to Rey's length distribution the sources of improbability are seen on analysis to be (*a*) the crowding up of seven eggs on 25·0 mm., (*b*) of 131 eggs on 22·0, and (*c*) of 63 on 22·5 mm. These contribute more than half the value of χ^2! In other words there is little reason to doubt that the variability in length of Cuckoo's eggs would follow a normal curve, were it not for Rey's tendency to heap up observations on the ·5 and ·0 millimetre groups*. It does not therefore seem worth while pursuing the distribution of variability further on the present observations. We see that it is quite possible that the normal curve would really suffice to describe the frequency. On the other hand, if the *gentes* theory be considered as established, we should naturally expect the heterogeneity to show itself in some deviations from any smooth curve of frequency.

In conclusion I must gratefully acknowledge the generous assistance of Prof. Karl Pearson in the preparation of this paper and in the statistical examination of the measurements obtained: indeed without his aid I should have been quite unable to perceive their significance, nor would this paper have been written.

* The evil of this heaping up may be easily seen if we take the first group 19·05—20·05 instead of 18·75—19·75 still grouping by mm. The mean will now be found to be 22·10 and the standard deviation 1·0014 instead of 22·27 and ·9450 respectively!

MISCELLANEA.

Notes on Statistical Processes.

By W. PALIN ELDERTON, *Actuary.*

I.

An Alternative Method of calculating the Rough Moments from the Actual Statistics.

Let s be the position of any term in the series and $f(s)$ the frequency corresponding, then the following table (page 375) explains itself.

Consider the line opposite the first term and we notice that the sum of the series is given by the first summation S_1 and that the second which we will call S_2 gives the first moment of the whole distribution about a vertical through a point situated at unit distance before the point corresponding to $f(1)$. Still considering only the first line we see that S_3 gives each function multiplied by $\frac{n(n+1)}{2}$ or $\frac{n^2+n}{2}$, i.e. $(m_2'+m_1')\div 2$ where m' is written for the rough moment. Similarly S_4 gives $\frac{n^3+3n^2+2n}{6}$ and S_5 gives $\frac{n^4+6n^3+11n^2+6n}{24}$, and hence

$$S_2=m_1',$$

$$S_3=\frac{m_2'+m_1'}{2},$$

$$S_4=\frac{m_3'+3m_2'+2m_1'}{6},$$

$$S_5=\frac{m_4'+6m_3'+11m_2'+6m_1'}{24}.$$

A numerical example is as follows.

TABLE II.

Function	1st sum	2nd sum	3rd sum	4th sum
239	524	978	1648	2587
157	285	454	670	939
93	128	169	216	269
29	35	41	47	53
6	6	6	6	6
524	978	1648	2587	3854

TABLE I.

No. of term s	Function $f(s)$	Sum of function from s to n	Sum of previous column from s to n (second sum)	Sum of previous column from s to n (third sum)	Sum of previous column from s to n (fourth sum)	Sum of previous column from s to n (fifth sum)
1	$f(1)$	$f(1)+f(2)+\ldots+f(n)$	$f(1)+2f(2)+\ldots+nf(n)$	$f(1)+3f(2)+\ldots+\frac{n(n+1)}{2}f(n)$	$f(1)+4f(2)+\ldots+\frac{n(n+1)(n+3)}{3!}f(n)$	$f(1)+5f(2)+\ldots+\frac{n(n+1)(n+2)(n+3)}{4!}f(n)$
2	$f(2)$	$f(2)+\ldots+f(n)$	$f(2)+2f(3)+\ldots+(n-1)f(n)$	$f(2)+3f(3)+\ldots+\frac{n(n-1)}{2}f(n-1)$	$f(2)+4f(3)+\ldots+\frac{(n-1)n(n+1)}{3!}f(n-1)$	$f(2)+5f(3)+\ldots+\frac{(n-1)n(n+1)(n+2)}{4!}f(n-1)$
$n-2$	$f(n-2)$	$f(n)+f(n-1)+f(n-2)$	$f(n-2)+2f(n-1)+3f(n)$	$f(n-2)+3f(n-1)+5f(n)$	$f(n-2)+4f(n-1)+10f(n)$	$f(n-2)+5f(n-1)+15f(n)$
$n-1$	$f(n-1)$	$f(n)+f(n-1)$	$f(n-1)+2f(n)$	$f(n-1)+3f(n)$	$f(n-1)+4f(n)$	$f(n-1)+5f(n)$
n	$f(n)$	$f(n)$	$f(n)$	$f(n)$	$f(n)$	$f(n)$

It is unnecessary to form the fifth sum in detail as the total of the fourth gives the only term required. The totals check the work throughout. The rough moments (not reduced to unit frequency) from the equations given above are 978, 2318, 6612 and 21458 which agree with the result we should obtain by direct calculation.

If the rough moments about the mean are required the following is a convenient form :

$$\nu_2 = 2S_3/S_1 - d(1+d),$$
$$\nu_3 = 6S_4/S_1 - 3\nu_2(d+1) - d(1+d)(2+d),$$
$$\nu_4 = 24S_5/S_1 - 2\nu_3\{1+2(d+1)\} - \nu_2\{6(1+d)(2+d)-1\} - d(1+d)(2+d)(3+d),$$

where d is the distance between the mean and the point used for calculating the untransferred rough moments. Obviously $d = S_2/S_1$.

I find that if there are few terms to be dealt with the direct method is quicker but for longer series the above has a distinct advantage. The summation method is really due to Mr G. F. Hardy, but though he has remarked that it is practically the same as the method of moments, I have not traced any previous note of the actual connection between the two methods as statistical processes, though Professor Pearson tells me he believes the summation method of reaching moments is given by Coradi in a paper on the Abdank-Abanowitz integrator, and J. Massau, "Mémoire sur l'intégration graphique et ses applications," Paris, 1887, gives the connection between successive integrations and the moments. I understand that the integraph does not give very satisfactory results for the higher moments, but though the method is like the one with which we are dealing, the latter is clearly not open to the same objection.

The method can be extended to enable us to deal with correlation tables. In order to find the coefficient of correlation we require the means and standard deviations of the x's and y's and the xy-moment. The mean and S. D.'s can be found by treating the totals of the x-columns or y-rows in the same way as we did the entries in Table II. but a quicker method can be devised, which however will be easier to explain with the help of an example.

TABLE III. *Correlation Table.*

		x					Totals
		1	*2*	*3*	*4*	*5*	
y	*1*	2	6	10	20	3	41
	2	1	5	30	9	2	47
	3	—	9	28	30	7	74
	4	1	—	11	16	10	38
Totals		4	20	79	75	22	200

We now obtain from this table another (Table IV.) in the same form giving the y-sum of it by summing each column continuously, and then obtain Table V. by summing Table IV. across continuously.

TABLE IV. *y-sum of Table III.*

		x					Totals
		1	*2*	*3*	*4*	*5*	
y	*1*	4	20	79	75	22	200
	2	2	14	69	55	19	159
	3	1	9	39	46	17	112
	4	1	—	11	16	10	38
Totals		8	43	198	192	68	509

TABLE V.

x-sum of Table IV., i.e. a table giving all cases for x, y group and over in Table III.

		x					Totals
		1	*2*	*3*	*4*	*5*	
y	*1*	200	196	176	97	22	691
	2	159	157	143	74	19	552
	3	112	111	102	63	17	405
	4	38	37	37	26	10	148
Totals		509	501	458	260	68	1796

The totals in the right-hand column of Table IV. give the first sum of the total in the right-hand column of Table III. and are the same as the column $x=1$ in Table V. The total of Table III. or the sum of col. 1 in Table IV. gives the mean of the y's $(509 \div 200)$; similarly the sum of the row $y=1$ gives the mean of the x's $(691 \div 200)$.

The total of Table V. gives the xy-moment (1796) and the x-standard deviation is found by forming from the first row the series 691, 491, 295, 119, 22 and summing it, i.e. 1618. The second moment about the mean can then be found; the numerical working being as follows:

$$x\text{-mean} = 691 \div 200 = 3{\cdot}455,$$

$$\nu_2 = 2S_2/S_1 - d(1+d) \text{ by formula given above}$$

$$= \frac{1618 \times 2}{200} - 3{\cdot}455 \times 4{\cdot}455$$

$$= {\cdot}788.$$

Similarly with the y-moments

$$y\text{-mean} = 509 \div 200 = 2{\cdot}545,$$

$$\nu_2 = \frac{(509+309+150+38) \times 2}{200} - 2{\cdot}545 \times 3{\cdot}545$$

$$= 1{\cdot}038.$$

$$\text{The } xy\text{-moment} = \frac{1796}{200} - 2{\cdot}545 \times 3{\cdot}455$$

$$= {\cdot}1870,$$

$$r = {\cdot}1870 \div \sqrt{{\cdot}788 \times 1{\cdot}038}$$

$$= {\cdot}207.$$

In this example the moments have not been adjusted as the totals of the rows did not seem to warrant the assumption of high contact.

Table V. can be formed directly from Table III. but it is a little troublesome to do so at first. The method can best be understood by seeing how any item is made up; thus

$$63 = 30+17+26-10, \quad 97 = 20+74+22-19, \quad 102 = 28+63+37-26 \text{ and so on.}$$

It is not difficult to see how Table V. succeeds in giving the xy-moment; we have merely to remember the numbers by which the various terms in a correlation table have to be multiplied to produce the required result. The final item in Table III. for instance has to be multiplied by 20. It occurs in every item in the $x=5$ column of Table IV. and therefore in every item of Table V.; there are 20 items. Similarly the entry 28 in Table III. has to be multiplied by 9;

it occurs in $x=3$ and $y=1, 2$ and 3 in Table IV. and consequently in all the first 9 groups of Table V. and it is therefore correctly treated.

I have found when dealing with correlation tables that the summation method saves time when the table is fairly large. The method might be extended to simplify some of the calculations which are required when non-linear regression is investigated (see Professor Pearson's Drapers' Research Memoir, Biometric Series, II. *On Skew Correlation*).

II.

The Application of Certain Quadrature Formulae.

The usual quadrature formulae express an area in terms of the ordinates at the beginning and end of the base and a varying number of intermediate ordinates, but in some statistical work it is convenient to have values for the areas in terms of ordinates both within and without the base on which the area stands. Symbolically we have to express $\int_{-\frac{1}{2}}^{\frac{1}{2}} y\,dx$ in terms of y_0, y_1, y_{-1}, y_2, y_{-2}, etc.

Let $$y_x=a+bx+cx^2+dx^3+ex^4,$$

then $$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx=a+\frac{c}{12}+\frac{e}{80},$$

and $$y_0=a,$$
$$y_{-1}+y_{+1}=2(a+c+e),$$
$$y_{-2}+y_{+2}=2(a+4c+16e).$$

Now assume the required integral can be put in the form

$$hy_0+k(y_1+y_{-1})+l(y_2+y_{-2}),$$

substitute the values given just above and equate coefficients of a, c and e respectively to 1, $\frac{1}{12}$ and $\frac{1}{80}$ and we obtain

I. $$\int_{-\frac{1}{2}}^{\frac{1}{2}} y\,dx=\frac{1}{5760}\{5178y_0+308(y_1+y_{-1})-17(y_2+y_{-2})\}.$$

II. If $$y=a+bx+cx^2,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y\,dx=\frac{1}{24}\{y_{-1}+22y_0+y_1\}.$$

III. If $$y_x=a+bx+cx^2+dx^3+ex^4,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx=\frac{1}{5760}\{6463y_0-2092y_1+2298y_2-1132y_3+223y_4\}.$$

IV. If $$y_x=a+bx+cx^2+dx^3,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx=\frac{1}{24}\{27y_0+17y_1+5y_2-y_3\}.$$

V. If $$y_x=a+bx+cx^2,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx=\frac{1}{24}\{25y_0-2y_1+y_2\}.$$

VI. If $$y_x=a+bx+cx^2,$$
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx=\frac{1}{24}\{13(y_{-\frac{1}{2}}+y_{\frac{1}{2}})-(y_{-\frac{3}{2}}+y_{\frac{3}{2}})\}.$$

VII. If $$y_x = a + bx + cx^2 + dx^3 + ex^4,$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx = \frac{1}{1440}\{802\,(y_{-\frac{1}{2}} + y_{+\frac{1}{2}}) - 93\,(y_{-\frac{3}{2}} + y_{\frac{3}{2}}) + 11\,(y_{-\frac{5}{2}} + y_{\frac{5}{2}})\}.$$

VIII. If $$y_x = a + bx + cx^2 + dx^3 + ex^4,$$

$$\int_{-\frac{1}{2}}^{\frac{3}{2}} y_x dx = \frac{1}{90}\{29y_{-\frac{1}{2}} + 124y_{\frac{1}{2}} + 24y_{\frac{3}{2}} + 4y_{\frac{5}{2}} - y_{\frac{7}{2}}\}.$$

IX. If $$y_x = a + bx + cx^2,$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx = \frac{1}{12}\{5y_{-\frac{1}{2}} + 8y_{\frac{1}{2}} - y_{\frac{3}{2}}\}.$$

The most obvious purpose for which these formulae can be used in biometric work is for calculating the areas from the values of ordinates when the equation to the curve has been determined. It is clearly convenient to calculate as few ordinates as possible and I have found Formulae I. and II. especially convenient when expressed in the form of differences; thus II. can be written

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y\,dx = y_0 + \frac{1}{24}\{\Delta y_0 - \Delta y_{-1}\},$$

and the adjustment from the mid-ordinate is easily calculated.

Formula I. can be written

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y\,dx = y_0 + \frac{291}{5760}\{\Delta y_0 - \Delta y_{-1}\} - \frac{17}{5760}\{\Delta y_1 - \Delta y_{-2}\}$$

and the fractions are ·0505 and ·00295.

Formulae VI. and VII. are useful when the ordinates at the beginning and end of each group have been calculated.

A numerical example of the use of these formulae in a case where their accuracy can be tested will be of interest. We will assume that the ordinates of the normal curve have been calculated for the equidistant values of x and the areas corresponding to these as mid-ordinates are required.

TABLE VI.

x	Value of ordinate	Δ	Area by Formula I.	Area by Formula II.	True value
·80	·2896916	·0785785			
·40	·3682701	·0306722	·366223	·366274	·366218
·00	·3989423	− ·0306722	·396307	·396386	·396298
·40	·3682701	− ·0785785	·366223	·366274	·366218
·80	·2896916	− ·0955055	·288992	·288986	·288994
1·20	·1941861	− ·0832653	·194740	·194696	·194746
1·60	·1109208	− ·0569298			
2·00	·0539910				

The second and the last columns of this table were found from W. F. Sheppard's Tables, *Biometrika*, Vol. I. pp. 182 *et seq.* For frequency curve work we do not generally require a large number of figures, and as the total frequency does not often exceed 5000, Formula II. would be sufficiently accurate while Formula I. would give a satisfactory answer in almost all possible cases.

III.

Adjustment of Moments.

The quadrature formulae given above can be used very conveniently for adjusting the statistical moments.

We may first take the calculation of moments when the ordinates at equidistant points are known. Let $y_0, y_1, y_2 \ldots y_{n-1}$ be given and we require $\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x dx$. Now

$$\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx + \int_{\frac{1}{2}}^{\frac{3}{2}} y_x dx + \ldots + \int_{n-\frac{3}{2}}^{n-\frac{1}{2}} y_x dx.$$

If we now apply Formula I. we see that we can use it for all the integrals on the right-hand side of this equation except the first two and the last two and the value of these can be taken from Formula IV. Summing the values we have

$$\begin{aligned}\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x dx = \frac{1}{5760}\{&6463 y_0 + 4371 y_1 + 6669 y_2 + 5537 y_3 \\ &+ 5760 (y_4 + y_5 + \ldots + y_{n-5}) + 5537 y_{n-4} \\ &+ 6669 y_{n-3} + 4371 y_{n-2} + 6463 y_{n-1}\} \quad \ldots\ldots\ldots\ldots\ldots \text{ X.},\end{aligned}$$

which means that we can multiply the first and last ordinates by $\frac{6463}{5760}$ (or 1·122205), the second and last but one by $\frac{4371}{5760}$ (or ·758854), the third and last but two by $\frac{6669}{5760}$ (or 1·157813), the fourth and last but three by $\frac{5537}{5760}$ (or ·961285), leave all the other ordinates unaltered and work out the moments in the usual way from the modified series of ordinates which will now give the proper values for equating to the moments from the formula for the curve.

If there be high contact at each end of the curve—and ordinates are known—there is no reason why Formula I. should not be used even for the end groups, and this tells us that the rough statistical moments require no modification in such a case.

The above rules have been applied in actual cases, thus Formula VI. was found when tested, by using 12 ordinates to approximate to $\int_0^1 \frac{dx}{1+x}$, to give an error of $-\cdot 000000205$ which, though it is greater than the best formulae given by Professor Pearson in *Biometrika*, Vol. I. pp. 278, 279, is sufficiently accurate for almost all conceivable purposes. The rule in cases of high contact was tested by adding 12 ordinates of the normal curve calculated to 5 decimal places, which gave 1·24998 instead of 1·25000. A type III. curve which had high contact gave 24473 with 9 ordinates instead of 24475, so that in each case the result was practically exact. A further test can however be applied, for if we assume that areas and not ordinates are known we can apply the formulae and reach Sheppard's adjustments. For if areas are known we are given as the rough t^{th} moment the expression

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx\, X^t + \int_{\frac{1}{2}}^{\frac{3}{2}} y_x dx\, (X+1)^t + \int_{\frac{3}{2}}^{\frac{5}{2}} y_x dx\, (X+2)^t + \ldots\ldots + \int_{n-\frac{3}{2}}^{n-\frac{1}{2}} y_x dx\, (X+n-1)^t,$$

and we require $\int_{-\frac{1}{2}}^{n-\frac{1}{2}} (X+x)^t y_x dx$. The series of integrals can be written by the help of Formula I. as

$$\frac{1}{5760}\{\ldots + [5178 h^t + 308\{(h-1)^t + (h+1)^t\} - 17\{(h-2)^t + (h+2)^t\}] y_h + \ldots\},$$

where X is neglected in order to simplify, and working out this general coefficient we have

$$\frac{1}{5760}\{5760 h^t + 240t(t-1) h^{t-2} + 3t(t-1)(t-2)(t-3) h^{t-4} \text{ etc.}\}.$$

If $t=2$ this becomes h^2+12 and if $t=4$, $h^4+\frac{1}{2}h^2+\frac{1}{80}$. We have however already noticed that if there is high contact the value of $\int_{-\frac{1}{2}}^{n-\frac{1}{2}}(X-x)^t y_x\,dx$ is given by the sum of the ordinates, i.e. the second moment is given by a series of which the general term is h^2y_h and the fourth by one whose general term is h^4y_h; hence if μ be written for the true and ν for the unadjusted moment we have

$$\mu_2+\frac{1}{12}=\nu_2,$$

and
$$\mu_4+\frac{1}{2}\mu_2+\frac{1}{80}=\nu_4,$$

or
$$\nu_2-\frac{1}{12}=\mu_2,$$

and
$$\nu_4-\frac{1}{2}\nu_2+\frac{7}{240}=\mu_4.$$

The odd moments are seen to require no modification.

It is interesting to note that Formula II. will give the adjustment for the second moment but that for the fourth it gives $\nu_4-\frac{1}{2}\nu_2+\frac{1}{24}=\mu_4$ which only differs from the true result by $\frac{3}{240}$. To demonstrate Sheppard's correction for the nth moment a parabola of at least the nth order must be used.

There is another method of a very simple character by which adjustments when areas are given can be reached; it consists of finding ordinates from the given areas and then working on the values obtained.

It is easy to obtain formulae suitable for our purpose in the following way:

Let $y_x=a+bx+cx^2+dx^3+ex^4$, then

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} y_x dx=A_0 \quad =a+\frac{2c}{24}+\frac{2e}{160},$$

$$\int_{\frac{1}{2}}^{\frac{3}{2}} y_x dx=A_1 \quad =a+b\ +\frac{26}{24}c+\frac{80}{64}d+\frac{242}{160}e,$$

$$\int_{-\frac{3}{2}}^{-\frac{1}{2}} y_x dx=A_{-1}=a-b\ +\frac{26}{24}c-\frac{80}{64}d+\frac{242}{160}e,$$

$$\int_{\frac{3}{2}}^{\frac{5}{2}} y_x dx=A_2 \quad =a+2b+\frac{98}{24}c+\frac{544}{64}d+\frac{2882}{160}e,$$

$$\int_{-\frac{5}{2}}^{-\frac{3}{2}} y_x dx=A_{-2}=a-2b+\frac{98}{24}c-\frac{544}{64}d+\frac{2882}{160}e.$$

These equations can easily be solved and we obtain the following result:

$$a=\frac{1}{1920}\{2134A_0-116(A_{-1}+A_{+1})+9(A_{-2}+A_{+2})\} \quad \text{..... XI.},$$

$$b=\frac{1}{1920}\{\qquad -1360(A_{-1}-A_{+1})+200(A_{-2}-A_{+2})\},$$

$$c=\frac{1}{1920}\{-2640A_0+1440(A_{-1}+A_{+1})-120(A_{-2}+A_{+2})\},$$

$$d=\frac{1}{1920}\{\qquad 320(A_{-1}-A_{+1})-160(A_{-2}-A_{+2})\},$$

$$e=\frac{1}{1920}\{480A_0-320(A_{-1}+A_{+1})+80(A_{-2}+A_{+2})\}.$$

But $y_0 = a$, and if there be high contact we can form all the ordinates $y_0, y_1, y_2 \ldots\ldots y_{-1}, y_{-2}, \ldots$ by using the expression given for a, hence, if X be the distance of y_0 from the vertical about which moments are calculated we have as the general term for the nth moment

$$[2134X^n - 116\{(X-1)^n + (X+1)^n\} + 9\{(X-2)^n + (X+2)^n\}],$$

which after reduction becomes

$$\left[1920\,X^n - \frac{n(n-1)}{2!}.80.X^{n-2} + \frac{n(n-1)(n-2)(n-3)}{4!}.28.X^{n-4}\right],$$

and this will be found to give Sheppard's adjustments.

The values of b, c, d and e are given above as they may be useful in some cases other than those with which we are now dealing, e.g. for some work when parabolas are being used. They enable us to find, for instance,

$$y_{-2} = \frac{1}{1920}\{1689A_{-2} + 684A_{-1} - 746A_0 + 364A_1 - 71A_2\},$$

which, by the way, if applied to all terms of the series also give Sheppard's adjustments when high contact is assumed.

The method in a slightly modified form might be applied to enable us to deal with the cases in which the curve rises sharply from zero, but the difficulty is that in all such cases the actual starting point makes a very great difference to the result we obtain. In fact it is possible if we assume the curve to start at the beginning of one of the groups to obtain a negative ordinate at the middle point of the group and the result is in consequence extremely unsatisfactory, though it gives some idea of where the curve really starts.

Using the same notation as above, viz. $\int_{r-\frac{1}{2}}^{r+\frac{1}{2}} y\,dx = A_r$, and also the same method, the following formulae can be obtained :

If $y_x = a + bx + cx^2 + dx^3 + ex^4$,

$$y_0 = \frac{1}{1920}\{2134A_0 - 116(A_{-1} + A_1) + 9(A_{-2} + A_2)\} \text{ as above,}$$

$$y_1 = \frac{1}{1920}\{-71A_2 + 2044A_1 - 26A_0 - 36A_{-1} + 9A_2\} \ldots\ldots\ldots\ldots \text{XII.,}$$

$$y_2 = \frac{1}{1920}\{1689A_2 + 684 - 746A_0 + 364A_{-1} - 71A_{-2}\} \ldots\ldots\ldots\ldots \text{XIII.}$$

If $y = a + bx + cx^2$,

$$y_0 = \frac{1}{24}\{-A_{-1} + 26A_0 - A_1\} \ldots\ldots\ldots\ldots \text{XIV.,}$$

$$y_1 = \frac{1}{24}\{23A_1 + 2A_0 - A_{-1}\} \ldots\ldots\ldots\ldots \text{XV.}$$

In order to apply these formulae to the problem of correcting moments we first find the ordinates corresponding to the given areas by means of Formulae XI. to XV. and then use Formula X. or some similar simpler expression such as

$$\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x\,dx = \frac{1}{24}\{26y_0 + 21y_1 + 25y_2 + 24y_3 + 24y_4 + \ldots\} \ldots\ldots\ldots\ldots \text{XVI.,}$$

which is formed from equations II. and V. in order to find the terms to be used for giving the corrected moments. The exact area may not be reproduced, for if we find the first term y_0 by XV. and the other terms by XIV. and then use XVI. we have the following result:

$$\int_{-\frac{1}{2}}^{n-\frac{1}{2}} y_x dx = \frac{1}{24}\{26y_0+21y_1+25y_2+24y_3+\ldots\}$$

$$= \frac{1}{24}\times\frac{1}{24}\{26\times 23\Delta_0+26\times 2\Delta_1-26\Delta_2$$
$$-21\Delta_0+21\times 26\Delta_1-21\Delta_2$$
$$-25\Delta_1+25\times 26\Delta_2-25\Delta_3$$
$$-\text{etc.}\}$$

$$= \frac{1}{576}\{577\Delta_0+573\Delta_1+579\Delta_2+575\Delta_3+576\Delta_4+\ldots\},$$

or the area is overstated by

$$\frac{1}{576}\{\Delta_0-3\Delta_1+3\Delta_2-\Delta_3\}.$$

When a parabolic curve is being fitted these formulae can be applied as they stand, for in such cases we can generally choose our range to some extent, that is the formula itself does not fix an actual starting place for our curve; but in much frequency curve work the difficulty of adjusting the moments is, as has been already remarked, considerably increased owing to the rapid rise of the curve from zero. In such cases, especially when the terms are few in number or there seems a likelihood from the original statistics that the curve does not start at the beginning of the base of the first group, I think the formulae should be applied neglecting the first group, which should be examined separately.

Taking the following unpromising series (col. 1) from *Phil. Trans.* Vol. 197 A, pp. 454—456, the following table was made.

TABLE VII.

Frequency (1)	Ordinate by XIV. & XV.	Modified ordinate by XVI.
47	(neglected)	(neglected)
762	742·7	804·6
160·5	141·2	123·6
20	14·8	15·4
5	4·5	4·5
1·5	1·3	1·3
996		

If the curve starts at the middle of the base of the 47 group its mid-distance from the middle point of the base of the 762 group is really $\frac{3}{4}$ and this should be taken into account in calculating the moments. Now it is easy to see that the first group probably relates to a small base because the mid-ordinate calculated from $\frac{1}{24}(23\Delta_0+2\Delta_1-\Delta_2)$ is -8. If in order to

simplify matters we assume that half a base is the only alternative to a whole one the moments can be calculated as follows :

TABLE VIII.

Modified ordinate f	Distance d from 804·6 ordinate	fd	fd^2	fd^3	fd^4
47·0	−·75	−35·2	+26·4	−19·8	+14·8
804·6	—	—	—	—	—
123·6	1	123·6	123·6	123·6	123·6
15·4	2	30·8	61·6	123·2	246·4
4·5	3	13·5	40·5	121·5	364·5
1·3	4	5·2	20·8	83·2	332·8
996·4		137·9	272·9	431·7	1082·1

The series of numbers from which Professor Pearson found the graduated figures given in col. (1) of Table VII. was 1, 49, 798, 123, 18, 5, 2 ; but owing almost entirely to the differences of 2·6 between 18 and 15·4 and ·7 between 2 and 1·3 the totals that are obtained are 996, 131, 325, 473, 1393, which differ widely from those given above. The mean is very close in comparison to that obtained by using the figures as they stand in col. (1) Table VII. but the other moments are little better. I give the result of the attempt to find adjustments in this awkward case because it shows that the method can give a series of numbers very like that required to give a true result, but the small differences make the result unsatisfactory.